1975MNRAS.172..133S

Mon. Not. R. astr. Soc. (1975) **172**, 133–139.

A NON-AXISYMMETRIC RIGID ROTATOR MODEL FOR MAGNETIC STARS

M. J. Stift

Dunsink Observatory, Castleknock, Co. Dublin, Eire

(Received 1975 February 17; in original form 1975 January 31)

SUMMARY

A generalized, non-axisymmetric decentred dipole model is proposed to explain asymmetric magnetic variations and phase shifts between the respective extrema of effective field and surface field of magnetic stars. Detailed models are derived for two stars and it is shown that the model can reproduce the observed magnetic variations of all well-known magnetic stars but one.

1. INTRODUCTION

A detailed study of the magnetic variations of all stars with well-known magnetic variations (Stift 1974) has shown that Landstreet's (1970) decentred dipole model can satisfactorily explain the magnetic behaviour of most of these stars. This particular Oblique Rotator Model consists of a magnetic dipole situated near the equatorial plane of the stars with axis going through the centre of the star; the angle between rotational and magnetic axis is thus near 90°. The principal virtue of Landstreet's model compared with the centred dipole model (Stibbs 1950) is the much wider range in the variations of effective field H_e and surface field H_s (for definition see below). The magnetic variations of HD 65339, HD 112413 and HD 153882 however largely exceed the possible range in Landstreet's model. This indicates a more complex general field geometry of the Oblique Rotator which includes the decentred dipole model only as a special case. It can also be inferred from observations of H_e and H_s for HD 126515 and HD 137909, which show marked phase shifts between the respective field extrema of H_e and H_s, that this yet unknown general configuration is certainly non-axisymmetric. The intention of this paper is to investigate field geometrics which are non-axisymmetric, which include Landstreet's model, and which are characterized by the smallest possible number of parameters, i.e. generalized dipole field configurations.

2. A GENERALIZED RIGID ROTATOR MODEL

The most general dipole field configuration consists of a dipole placed at an arbitrary point inside the star and inclined at an arbitrary angle with respect to the rotational axis. It is immediately evident that Landstreet's model represents a special case of this general rigid rotator—the main axis of the dipole always goes through the centre of the stars. In the generalized model the axis of symmetry of the dipole usually does not intersect the surface of the star vertically, which means that phase shifts can occur between the respective field extrema of

$$H_e = \iint H_z I \, dA \Big/ \iint I \, dA \qquad (1)$$

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System

1975MNRAS.172..133S

and

$$H_s = \iint |\mathbf{H}|\, I\, dA \bigg/ \iint I\, dA \tag{2}$$

where H_z is the line-of-sight component of $\mathbf{H}$ and I is the limb-darkening function

$$I = 1 - k + k \cos\theta \quad (0 \leq \theta \leq \pi/2). \tag{3}$$

A value of $k = 0.45$ is generally assumed for Ap stars.

Thus this model can predict asymmetric variations in both H_e and H_s, accompanied by phase shifts between the respective field extrema. In addition it enlarges the range in the field extrema of Landstreet's model which is essential for the stars enumerated in *I*.

An arbitrary position of the dipole coordinate system relative to the star's rotational system can be regarded as the results of three rotations, the corresponding three angles being, e.g., the Eulerian angles. These three angles, together with the three coordinates which give the location of the dipole, and the angle between the rotational axis of the star and the line of sight, define the model. We can reduce the number of parameters if we take into account the fact that the dipole field is symmetric around the z-axis of the dipole. The model which is explained in detail below is defined by six parameters, the x-coordinate of the dipole being set to zero without loss of generality.

Let

$$S = S_\gamma S_\beta S_\alpha =$$

$$\begin{pmatrix} \cos\beta & \cos\alpha\sin\beta & \sin\alpha\sin\beta \\ -\sin\beta\cos\gamma & \cos\alpha\cos\beta\cos\gamma - \sin\alpha\sin\gamma & \sin\alpha\cos\beta\cos\gamma + \cos\alpha\sin\gamma \\ \sin\beta\sin\gamma & -\cos\alpha\cos\beta\sin\gamma - \sin\alpha\cos\gamma & -\sin\alpha\cos\beta\sin\gamma + \cos\alpha\cos\gamma \end{pmatrix} \tag{4}$$

be the transformation matrix from the rotational system of the star to the dipole system—S_α and S_γ are generating rotations around the x-axis and S_β a rotation around the z-axis—

$$S_\phi = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{5}$$

the matrix which generates the rotation of the dipole around the rotational axis, and

$$S_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{pmatrix} \tag{6}$$

the transformation matrix from the observer's system to the rotational system. If the coordinates of the dipole are given by the vector $\mathbf{x}$ and the coordinates of a surface point on the visible hemisphere by $\mathbf{z}$ we then obtain the coordinates of the surface point relative to the dipole system by

$$\mathbf{r} = S(S_\phi S_i \mathbf{z} - \mathbf{x}). \tag{7}$$

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System

1975MNRAS.172..133S

This can be used to calculate the field strength in the surface point in the dipole system by

$$\mathbf{h} = -\text{grad}\,(\mathbf{m}\cdot\mathbf{r}/r^3). \tag{8}$$

The magnetic field strength **H** in the observer's system is derived by

$$\mathbf{H} = S_i^{\mathrm{T}} S_\phi^{\mathrm{T}} S^{\mathrm{T}} \mathbf{h} \tag{9}$$

where S^{T}, S_i^{T}, S_ϕ^{T} are the respective transpose matrices of S, S_i, S_ϕ.

3. CALCULATION AND FITTING PROCEDURE

A FORTRAN IV program has been written by the author for the NOVA 2/10 computer of Dunsink Observatory. It calculates the variations of H_e and H_s for all possible sets of parameters by numerical integration over the visible hemisphere using double precision throughout.

The fitting procedure is similar to that given by Stift (1974) but up to five quantities are considered instead of three.

(a) The ratio of the effective maximum and minimum field

$$r = H_e(\max)/H_e(\min) \quad \text{where} \quad |r| \geq 1.$$

(b) The duration d, in terms of phase, during which H_e maintains the polarity of $|H_e|_{\max}$. If the field does not change polarity $d = 1{\cdot}0$.

(c) The range in the ratio $q = H_s/|H_e|_{\max}$ during all phases.

(d) The phase shifts $\Delta\phi_1$ between $H_s(\max)$ and $H_e(\min)$ and $\Delta\phi_2$ between $H_s(\min)$ and $H_e(\min)$ in terms of phase.

In making a fit the calculated values of all quantities must agree within the observational error range with the values deduced from a free-hand drawn curve of the observed magnetic variations. Preference is given to high obliquities, moderate displacements and small ranges of q whenever several different models give equally satisfactory results.

4. APPLICATION OF THE MODEL TO STARS WITH KNOWN SURFACE FIELDS

A generalized rigid rotator model can be fitted to the observed magnetic variations only if we dispose of a large number of observations of both effective field and surface field. This is the case for HD 65339, HD 126515 and HD 137909, the other three stars having unknown surface field strengths. The phase shifts observed between the variations in H_e and H_s for the former three stars indicate a considerable departure from axisymmetry, but with the exception of HD 65339 the range of the field variations can be explained by Landstreet's model. Thus HD 126515 and HD 137909 are excellent test-cases for the proposed rigid rotator. A detailed description of the analysis of these two stars follows.

HD 126515

Preston (1970) fitted a decentred dipole model to the magnetic variations of this star which predicts fairly accurately the range of H_e and H_s but which is in serious contradiction with the observed shape of the variation in H_e. Phase shifts of $\sim 0{\cdot}12$ phase units occur between $H_s(\min)$ and $H_e(\max)$ and of $\sim 0{\cdot}08$ phase units between

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System

1975MNRAS.172..133S

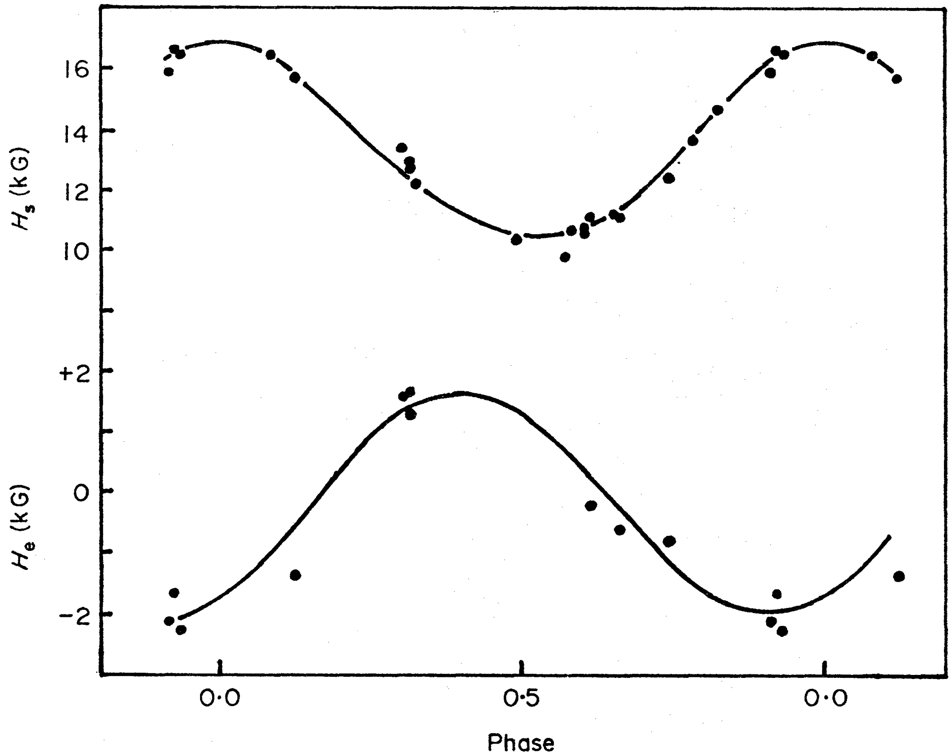

FIG. 1.

H_s(max) and H_e(min). The poor definition of the shape of the H_e variation makes it practically impossible to determine a value for the duration parameter d and leaves a large number of models compatible with the observations. The adoption of a value of d = 0·50–0·55 which seems the most probable, and a decentring parameter near to Preston's value, yields a model whose parameters are listed in Table I. Fig. 1 shows the excellent agreement between model and observations.

HD 137909

The magnetic behaviour of this star is somewhat similar to that of HD 126515. Wolff & Wolff (1970) carried out a detailed analysis of the magnetic variations and

TABLE I

Generalized decentred dipole models for two magnetic stars

Star	HD 126515	HD 137909
i (°)	20	14
α (°)	−93	−80
β (°)	100	103
γ (°)	−40	40
x_1	0·0	0·0
x_2	0·30	−0·23
x_3	0·10	−0·40
r	−1·25	−1·84
q	5·08–8·23	4·25–5·22
d	0·526	0·616
$\Delta\phi_1$	0·596	0·400
$\Delta\phi_2$	0·620	0·358
Reference	Preston (1970)	Wolff & Wolff (1970)

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System

1975MNRAS.172..133S

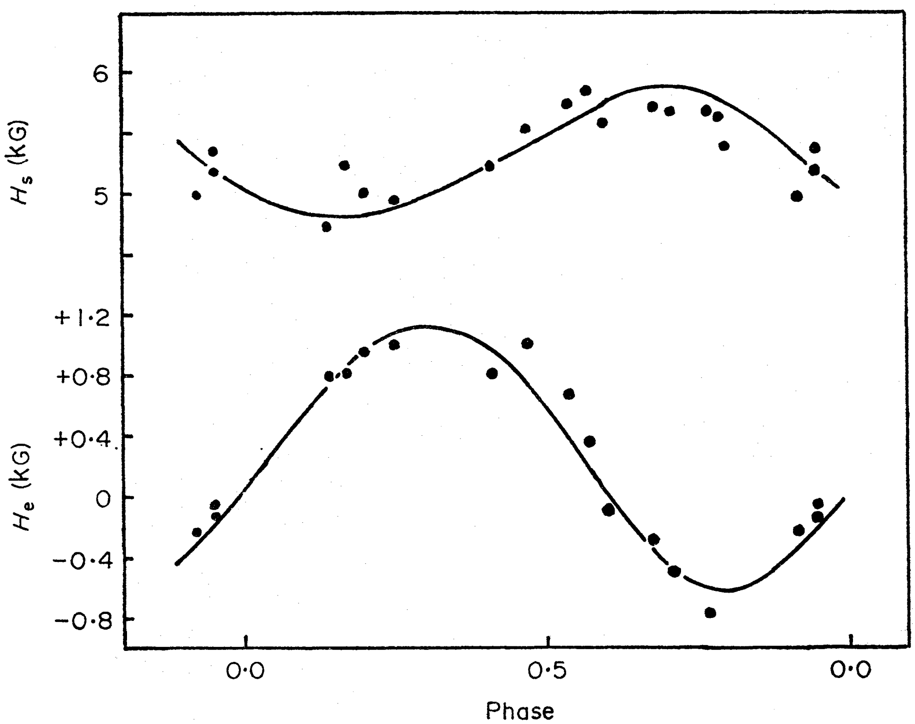

FIG. 2.

concluded that the decentred dipole model could well explain the observed ranges of H_e and H_s. But phase shifts occur between H_e and H_s variations and the proposed model of Wolff & Wolff only gives a very poor fit to the shape of the H_e variations. The scatter of the observations in H_s is considerable and makes the determination of the quantity q uncertain. However, the unusually large value of $d \sim 0{\cdot}60$ greatly reduces the number of possible models. The finally adopted model is listed in Table I. Fig. 2 shows the observed and calculated magnetic variations of this star.

HD 65339

The outstanding magnetic variations of this best-known magnetic star cannot be reconciled with any generalized decentred dipole model. It seems difficult to conceive any field geometry that could reproduce the sharp maximum in H_e without largely exceeding the observed values of H_s.

5. A SIMPLIFIED MODEL FOR STARS WITH UNKNOWN SURFACE FIELDS

The problem for HD 112413, HD 125248 and HD 153882 is slightly different. No measurements of H_s exist for these stars, so we are obliged to make certain assumptions about the variations in H_s. The ratio q_{max}/q_{min} does not exceed the value of $\sim 1{\cdot}8$ for the stars observed so far; it seems therefore safe to limit its value to < 2 for all stars. We also have to take into account that the phase shifts are unknown and we can set them arbitrarily to zero. This implies that the matrices S_β and S_γ reduce to unity matrices and that the new rigid rotator model is defined by four parameters. The model corresponds to Landstreet's model but for the

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System

1975MNRAS.172..133S

variable inclination of the dipole in the plane defined by rotational axis and dipole-position. This extra degree of freedom enables a wider range of field extrema, moderate variations in H_s are possible even for high inclinations, high obliquities and large displacements.

HD 112413

Pyper (1969) carried out a harmonic analysis of magnetic, spectrum and radial velocity variations of this star. Her results are satisfactory for the H_e variations but yield larger variations in H_s than hitherto observed in other magnetic stars. The main difficulty for any model, as in the case of HD 65339, is the sharp maximum which corresponds to a value of $d = 0{\cdot}38$–$0{\cdot}40$. It is possible to derive a model which fits the variations in H_e and which yields $q_{max}/q_{min} < 2$. The model parameters are listed in Table II.

TABLE II

Simplified models for three magnetic stars

Star	HD 112413	HD 125248	HD 153882
i (°)	42	50	40
α (°)	−90	−82	−92
x_1	0·0	0·0	0·0
x_2	−0·30	−0·25	−0·25
x_3	−0·70	−0·70	−0·60
r	−1·04	−1·17	−1·21
q	1·83–3·41	1·70–2·97	1·75–3·05
d	0·382	0·646	0·434
Reference	Pyper (1969)	Babcock (1960)	Babcock (1960), Hockey (1971)

HD 125248

The outstanding feature of the magnetic variations of this star is the exceptionally long duration of positive polarity combined with a low absolute value of r. Landstreet's model seems to be inadequate as all models which fit the H_e variations yield very large variations in H_s. Several four-parameter models can be found; preference is given to small displacements and small variations in H_s. The parameters of the finally adopted model are listed in Table II.

HD 153882

Measurements of effective field variations were made by Babcock (1960) and Hockey (1971). The curves agree reasonably well and both are used to derive a rigid rotator model. The difficulties encountered in the analysis of the variations in terms of Landstreet's model can be overcome with the four-parameter model. The parameters giving the best fit are listed in Table II.

6. CONCLUSIONS

The results in the preceding paragraph show that the proposed rigid rotator model may be adequate for almost all magnetic stars. Although the results are very encouraging for HD 126515 and HD 137909 and fairly satisfactory for HD 112413, HD 125248 and HD 153882 to which Landstreet's model cannot be

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System

1975MNRAS.172..133S

applied, it would be premature to conclude that the model is the right one for the three latter. Without measurements of H_s the model can neither be accepted as final nor discarded. With the necessary caution we can nevertheless draw the following conclusions:

(a) The generalized decentred dipole model seems to be applicable to all magnetic stars with the exception of HD 65339.

(b) The model can reproduce asymmetric curves of magnetic variation and phase shifts between the respective extrema of H_e and H_s, as observed in HD 126515 and HD 137909.

(c) The results in Tables I and II indicate that for the stars investigated the x_2-coordinate of the dipole never exceeds 0·35 units of radius whereas the x_3-coordinate can be as large as 0·7 units of radius. The conclusion that the distance of the dipole from the centre in Landstreet's model does not exceed 0·4 units of radius applies only to the x_2-coordinate in the generalized model.

(d) As in Landstreet's model the obliquity, i.e. the angle between rotational axis and dipole axis, always shows values near 90°.

REFERENCES

Babcock, H. W., 1960. Stellar magnetic fields, *Stellar atmospheres*, p. 282, ed. J. L. Greenstein, University Chicago Press, Chicago.
Hockey, M. S., 1971. *Mon. Not. R. astr. Soc.*, **152**, 97.
Landstreet, J. D., 1970. *Astrophys. J.*, **159**, 1001.
Preston, G. W., 1970. *Astrophys. J.*, **160**, 1059.
Pyper, D. M., 1969. *Astrophys. J. Suppl.*, **18**, 347.
Stibbs, D. W. N., 1950. *Mon. Not. R. astr. Soc.*, **110**, 395.
Stift, M. J., 1974. *Mon. Not. R. astr. Soc.*, **169**, 471.
Wolff, S. C. & Wolff, R. J., 1970. *Astrophys. J.*, **160**, 1049.

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System

1975MNRAS.172..133S

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System