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Zeeman Doppler maps: the true and the spurious*

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ABSTRACT

Numerical models of atomic diffusion in magnetic atmospheres of ApBp stars predict abundance structures that differ from the empirical maps derived with (Zeeman) Doppler mapping (ZDM). Whereas both equilibrium abundance stratification calculations and stationary solutions to the time-dependent diffusion equations predict (warped) rings about the magnetic equator in dipole-like magnetic geometries, spot-like structures dominate published abundance maps. An in-depth analysis of this apparent disagreement investigates the detectability by means of ZDM of a variety of abundance structures, including (warped) rings predicted by theory, but also complex spot-like structures. As it turns out, a number of published Doppler maps have to be considered spurious either because strong magnetic fields have been neglected or because they are based on spectra where photon noise dominates over the signal of the alleged abundance structures. Even when spectra of high signal to noise ratio are available, it can prove difficult or altogether impossible to correctly recover shapes, positions and abundances of a mere handful of spots, notwithstanding the use of all 4 Stokes parameters and an exactly known field geometry; the recovery of (warped) rings can be equally challenging. Inversions – based on just one or two spectral lines – of complex abundance maps usually admit of multiple solutions. It turns out that it is by no means guaranteed that a properly chosen regularisation function will lead to the true abundance map instead of some spurious one. Attention is drawn to the need for a study that would elucidate the relation between the stratified, field-dependent abundance structures predicted by diffusion theory on the one hand, and empirical maps obtained by means of "canonical" ZDM, i.e. with mean atmospheres and unstratified abundances, on the other hand. Finally we point out difficulties arising from the 2D nature of the atomic diffusion process in magnetic ApBp star atmospheres.

Key words. stars: abundances — stars: atmospheres – stars: chemically peculiar – stars : magnetic field – line: profiles

1. Introduction

Hardly could Ludendorff (1906) have imagined that his discovery of spectrum variations in the star α^2CVn (after which a whole new class of variables was named) would lead to so many exciting observational discoveries and to great progress in the understanding of stellar atmospheres. When Belopolsky (1913) not only found changes in width and depth of the spectral lines, but also signs of line doubling, he couldn't have anticipated the continuing, unabated interest in α^2CVn which has led to no less than 3 major studies over the past 13 years, Silvester et al. (2014) being the latest in the series. Babcock (1949b) and later Babcock & Burd (1952) observed a variable magnetic field in α^2CVn which could, as in the case of a number of other magnetic stars (Babcock 1958) be interpreted in terms of the so-called oblique rotator model where the magnetic axis of the rotating star is inclined with respect to the rotational axis (Babcock 1949a). Deutsch (1956) speculated that the observed periodic spectral variations and radial velocity variations were in some way related to the magnetic field and worked out a method to map the abundances of various chemical elements. This method was applied to α^2CVn by Pyper (1969) who derived curves of constant equivalent width of iron-peak elements and of rare-earth elements in addition to a magnetic field geometry made up of a dipolar and a quadrupolar contribution.

This early work credibly established the oblique rotator model and made it clear that there had to be some correlation between the strong magnetic fields of a number of Ap stars and the abundance anomalies seen in the spectra. Now that full Stokes $IQUV$ profiles of high S/N ratio are available at high spectral resolution, the simultaneous determination of the horizontal abundance distributions of various chemical elements and of the magnetic field geometry have become feasible (see for instance the study of α^2CVn by Silvester et al. 2014). Claims concerning the detection of vertical stratifications of chemical elements in the atmospheres of ApBp peculiar stars have been around for quite a while – see the review by Ryabchikova (2008) – but combined maps of horizontal and vertical element distributions in conjunction with empirical magnetic geometries have not yet appeared in the literature.

Regarding theory, the idea of atomic diffusion driven by radiative accelerations being responsible for the abundance anomalies found in ApBp stars is due to Michaud (1970). The role of magnetic fields has first been explored by Vauclair et al. (1979) who demonstrated the important effect of horizontal magnetic field lines on the accumulation of silicon. Much more recently, Alecian & Stift (2010), Stift & Alecian (2012) and Alecian (2015) have modelled the vertical distributions of several chemical elements in magnetic ApBp star atmospheres as a function of field angle and field strength. In addition to these equilibrium stratifications (with diffusive particle flux zero throughout the atmosphere) there are the time-dependent atomic diffusion calculations by Stift & Alecian (2016) which for almost horizontal field reveal surprisingly large over-abundances in high

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layers at relatively low field strengths. These theoretical predictions can be (and sometimes have been) confronted with empirical abundance maps derived with the help of (Zeeman) Doppler mapping (ZDM); for a very readable introduction to ZDM see Vogt et al. (1987). Generally, these comparisons have not resulted in agreement between theory based predictions and the detailed surface abundance distributions of a given star, with the blame put squarely and exclusively on theory. Alecian (2015) has presented a thorough discussion of the limitations to the current modelling of atomic diffusion, but a similar critical assessment of ZDM and the ability of its algorithms to detect complex abundance structures, in particular those predicted by diffusion theory, has yet to find its way into print.

There is thus a definite need to throw new light on this longstanding problem. In sec. 2 arguments are brought forward that (Z)DM maps might not be as credible as generally assumed, be it on account of neglected strong magnetic fields, impossibly high over-abundances, or noisy spectra. The consequences of the latter are discussed in sec. 3. A new Zeeman Doppler mapping code CossamDoppler is introduced in sec. 4 and successfully subjected to tests similar to those carried out on their Invers family of codes by Kochukhov & Piskunov (2002) and Kochukhov (2014a). CossamDoppler is shown to perform as well as the Invers codes as far as abundances are concerned. In sec. 5, zero-field inversions of stars with strong magnetic fields are shown to result in spurious maps, most definitely so in the case of HD 3980 (Nesvacil et al. 2012). Assuming for our tests that *all* stellar parameters are perfectly known, from atmosphere, inclination and rotational velocity to the *exact* magnetic field geometry, and using zero-noise spectra throughout makes it possible to deal exclusively with the problem of the determination of horizontal abundance distributions and to avoid interference from other unknown stellar parameters. Sec. 6 deals with the uncertainties in the maps recovered with 1 or 2 lines only: many solutions turn out to be non-unique, even with all four Stokes parameters modelled to a high degree of accuracy. Theoretical results which indicate that abundance stratifications depend on size and direction of the local magnetic field vector are shortly discussed in sec. 7, predicted (warped) rings and their detectability are the topic of sec. 7.1. 2D effects discussed in sec. 8 introduce further complications to the modelling of magnetic ApBp star atmospheres and to the interpretation of (Z)DM results. From the assembled evidence against a number of Doppler maps and in view of the unphysical assumptions underlying (Z)DM we conclude that the discrepancies between (Z)DM abundance maps and theoretical predictions are to be traced back to shortcomings of (Z)DM rather than solely to an alleged *"lack of up-to-date theoretical models"* as done by Nesvacil et al. (2012) .

2. (Zeeman) Doppler maps and theoretical predictions

Zeeman Doppler mapping (ZDM) has established itself over the last decades as a popular and apparently successful method for the mapping of stellar surface magnetic fields and of the abundance anomalies found in magnetic stars. It is still worthwhile to peruse Vogt et al. (1987) (=VPH) for a description of this method as applied to abundance mapping of non-magnetic stars. In addition to an overview of first efforts to map equivalent widths of various lines, this paper explains how to determine the spatial distribution of the abundances of different chemical elements. To constrain the ill-posed inversion problem, VPH impose the condition that the resulting map show maximum entropy. Their algorithm has been subjected to the famous "Vogtstar" test based on the 4 black letters "VOGT" written onto the white surface of a rotating sphere; these 4 letters are recovered by their algorithm in a recognisable way, as are 7 perfectly black spots in a further test.

Somewhat later, Piskunov & Kochukhov (2002) and Kochukhov & Piskunov (2002) (= KP02) gave details on how to carry out simultaneous Doppler mapping of stellar magnetic fields and of surface abundance inhomogeneities. Instead of maximum entropy regularisation, these authors are using Tikhonov regularisation in their Invers family of codes. However, according to Piskunov (2001), the exact form of the regularisation function is not important when there are sufficient observational data with small errors. Since that time, many magnetic Ap stars have been analysed by means of (Zeeman) Doppler mapping, resulting in abundance maps of CNO, Mg, Si, a number of iron-peak elements, and of rare earths. Wherever Stokes V profiles were available, or ideally the complete Stokes QUV polarised profiles, magnetic maps could be added to the abundance maps.

As it turns out, all the published results of (Z)DM seem to be at variance with the predictions of numerical diffusion models, whether based on equilibrium stratifications or on stationary solutions of the time-dependent case. From the calculations of Alecian (2015) and Stift & Alecian (2016), we expect for example Ni to behave very much like Fe but the respective maps derived for HD 50773 (Lüftinger et al. 2010a) differ considerably between each other, with a contrast in Ni of only 0.5 dex and a contrast in Fe of 3.4 dex. An over-abundant Fe region stretches almost all around the stellar equator (similar to Cr), whereas Ni is concentrated in 2 spots near the magnetic equator, Ti forming spots at the magnetic poles. Such major differences between these elements are not predicted by theory. Another case of gross differences between empirical results and theory is found in Nesvacil et al. (2012) who claim that in HD 3980 Si becomes as abundant as hydrogen in some spots, and Mn and O as abundant as He. Please note that in *all* ZDM analyses published so far, both over-abundances (even extreme ones) and under-abundances are assumed to remain constant throughout the atmosphere; in other words, stratified abundances are not considered.

Only by looking closely at *both* diffusion theory and (Z)DM can it become possible to reach an understanding of the reasons for the apparent general disagreement, perhaps even to reconcile the 2 worlds. There are excellent arguments against the validity of the abundance maps of HD 3980 (NL) which have been obtained by means of simple Doppler mapping, neglecting the influence of a magnetic fields of an estimated B_p = 7 kG polar field strength. Similarly, the results for HD 50773 (Lüftinger et al. 2010a) certainly are not above suspicion since it remains doubtful whether the signal of a mere 0.5 dex contrast in the Ni distribution can be detected in spectra of a mean S/N ratio of 120 – similar considerations certainly apply to Cu. It could prove of some relevance that the magnetic map of HD 24712 (Lüftinger et al. 2010b) which features a range in the field modulus of 2.1 − 4.2 kG, has been revised downwards by Rusomarov et al. (2015) who now claim 1.2 – 3.6 kG and a somewhat different geometry; remarkably enough, the abundance contrast of Nd iii has climbed from a modest 1.1 dex to a remarkable 3.5 dex. Yet the errors in the latest fits to the Stokes Q profiles of the Nd iii transitions continue to reach an uncomfortable 2% for amplitudes of at most 5%.

At this point we want to make it absolutely clear that it is in no way a goal of this paper to look at the problem of recovering magnetic field geometries. The focus is exclusively on the re-

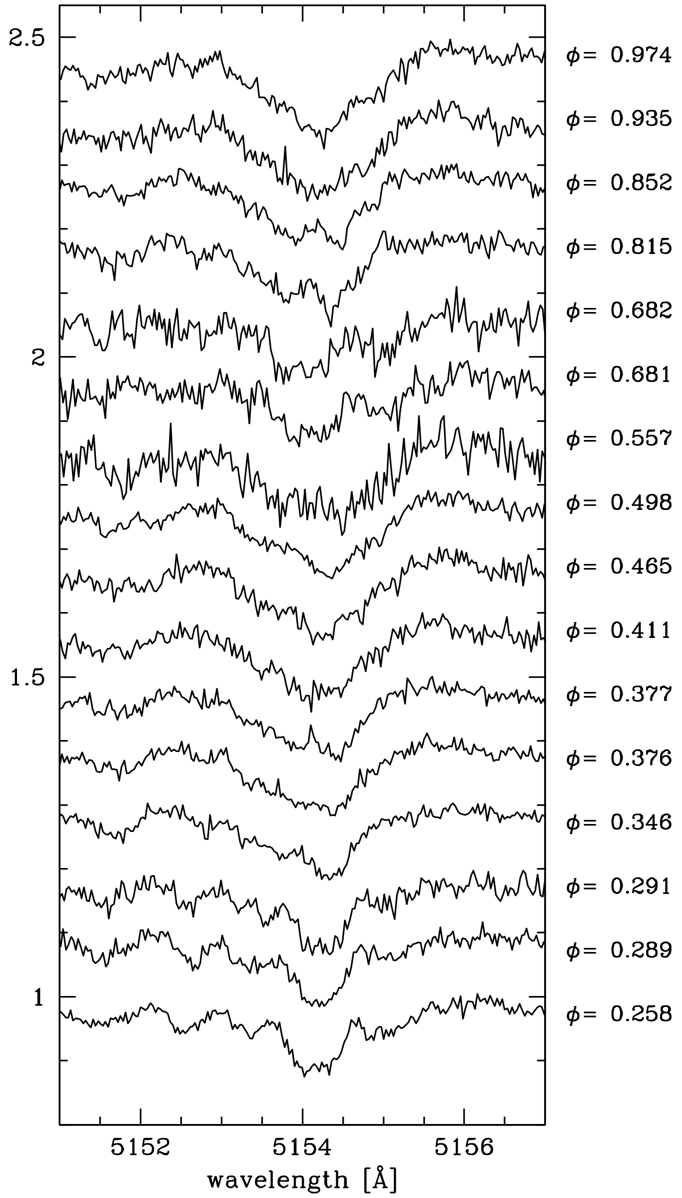

Fig. 1. The spectral interval 5151 – 5157 Å of the Narval spectra used by Lüftinger et al. (2010a) in the Doppler mapping study of HD 50773.

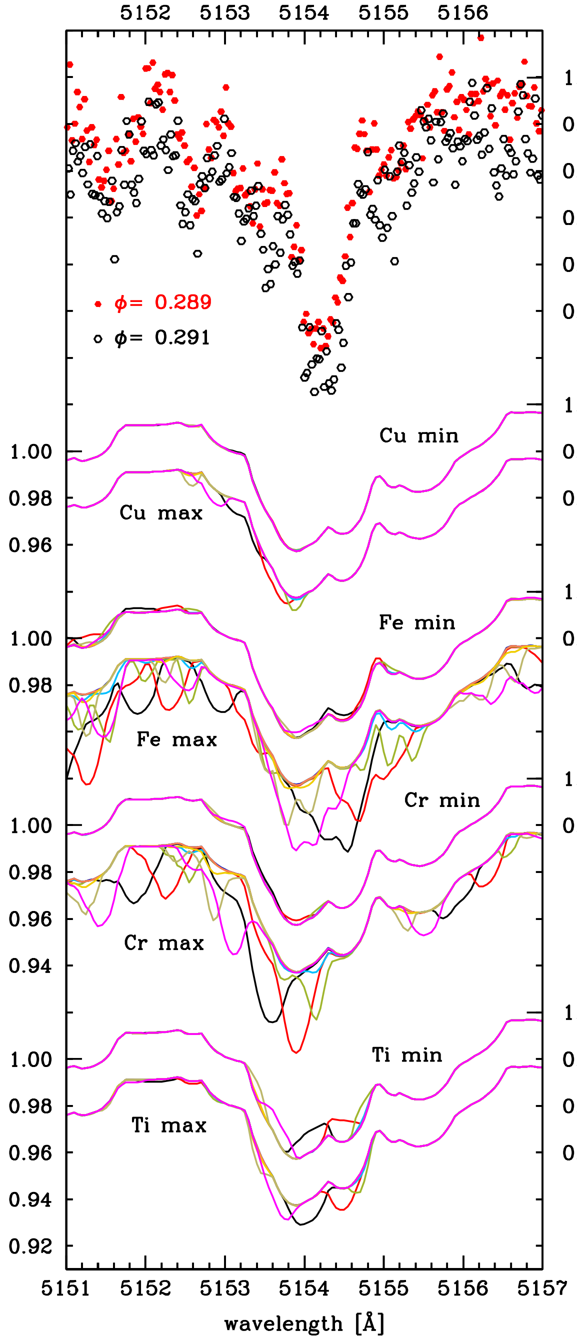

Fig. 2. Spectral variations with phase due to abundance spots of Ti, Cr, Fe and Cu. The spot at 20° latitude, with a radius of 20°, is assumed to feature in turn the respective maximum and minimum abundances of each of the four elements in question, with the other 91 elements exhibiting solar abundances.

covery of arbitrary abundance distributions. In this context, "arbitrary" means that abundance inhomogeneities need not necessarily have to take the shape of spots, and that they will in general not be "monolithic", i.e. with a constant abundance all over the structure/spot. One has to keep in mind that all the tests concerning abundance inhomogeneities to be found in the literature on (Zeeman) Doppler mapping are based on the detection of simple "monolithic" and mostly symmetric abundance spots. In VPH for example, the spots are perfectly black (intensity 0) against a perfectly white background (normalised intensity 1). In KP02, both the single spot and the triad of spots are "monolithic" with a vertically and horizontally constant 1.5 dex overabundance. Still, a look at the figs. 3, 5 and 9 of KP02 and in particular at p. 40 of Kochukhov (2014b) reveals that the mapping errors in the spots reach values of up to 0.6 dex, in accord with T. Ryabchikova's findings (private communication) that in ZDM, abundances could in places be wrong by up to 1 dex.

3. HD 50773 : noisy spectra and spurious maps

Lüftinger et al. (2010a) (= LU10) have carried out a Doppler mapping analysis of HD 50773, making it to the cover page of A&A with their colour maps of a number of metals, ranging from Mg to Cu. These maps have appeared a number of times since in reviews and lectures (e.g. Kochukhov 2014b) and therefore certainly figure among the Doppler maps best known to the general astronomical public. Their paper can also be considered the most unusual one as it is the only major article in the field of (Z)DM that does not feature a single plot of the observed inten-

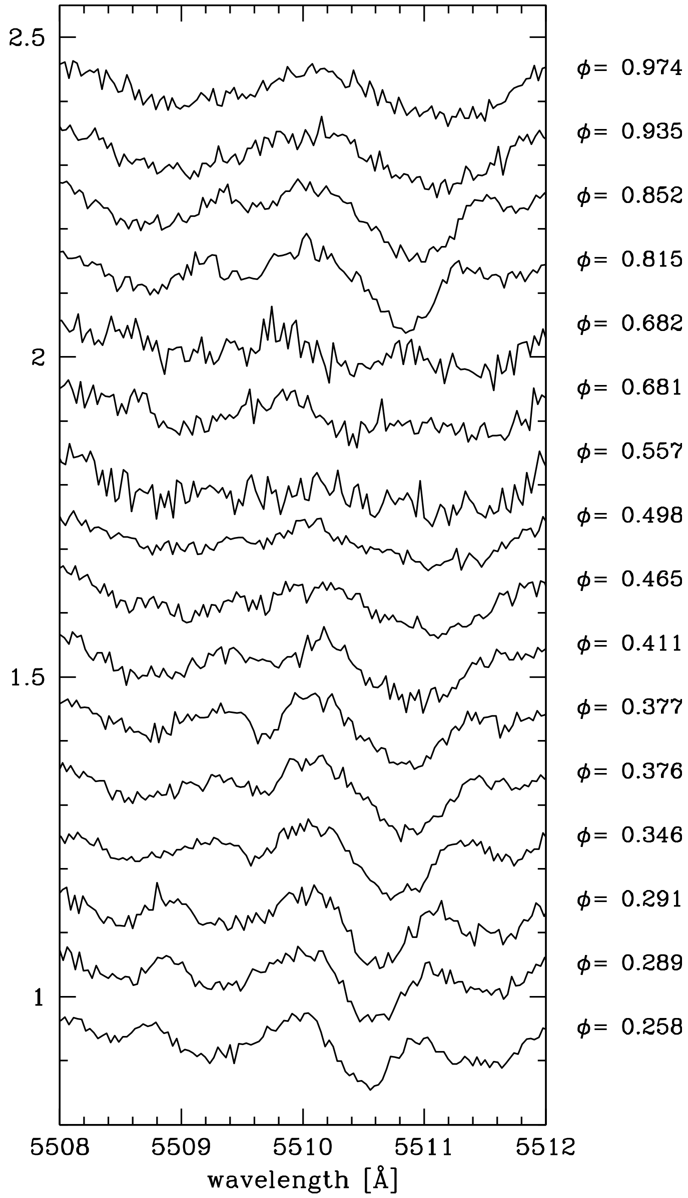

Fig. 3. The spectral interval 5508 – 5512 Å of the Narval spectra used by Lüftinger et al. (2010a) in the Doppler mapping study of HD 50773.

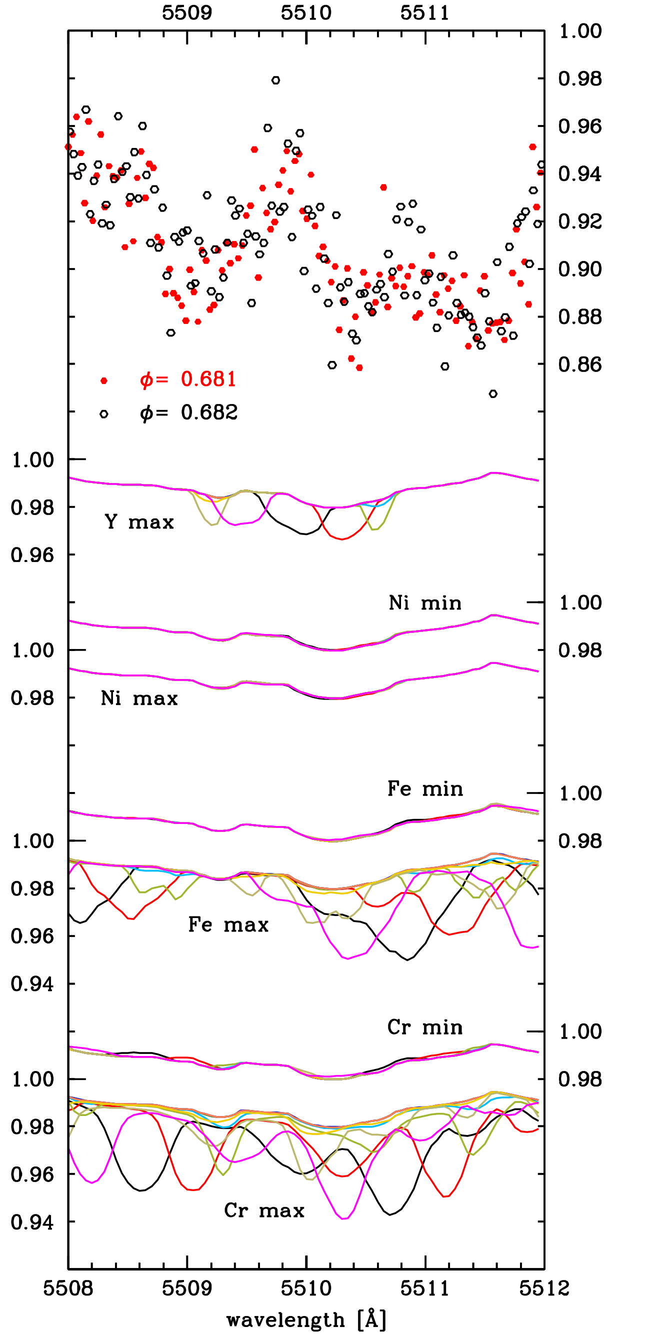

Fig. 4. Spectral variations with phase due to abundance spots of Cr, Fe, Ni and Y. The spot at 20° latitude, extending over a radius of 20°, is assumed to feature in turn the respective maximum and minimum abundances of each of the 4 elements in question, with the other 91 elements exhibiting solar abundances.

sity spectra and of the fits to these. According to table 1 of LU10, the *peak* S/N ratio of the spectra used in the inversion ranges from 64 to 170, with a mean value of about 120. Despite this poor data quality, LU10 have published maps for elements like Ni and Cu, derived from just 1 line each, with maximum abundance contrasts of only 0.5 dex and 1.1 dex respectively. Please note that the Ni I λ 5510.003 line is heavily blended with Cr, Fe and Y, the Cu II λ 5153.230 line with Cr, Ti and Fe. This makes HD 50773 a natural choice for a reassessment of the limits to the capabilities of (Z)DM.

Figs. 1 and 3 show the intervals 5151–5157 Å and 5508–5512 Å respectively of the 16 Narval spectra used by LU10, making one wonder how the abundance signatures on which all those detailed Doppler maps have been built could possibly have been extracted amid such photon noise. To visualise the nature of the problem, one simply has to compare pairs of spectra made at almost the same phase, as for example at $\phi = 0.289, 291$, $\phi = 0.376, 377$ and $\phi = 0.681, 682$. It is obvious that a statistically optimum fit to the profile at phase 0.289 will not really fit the profile at phase 0.291 (see the top of Fig. 2). The same holds true for the λ 5510 blend, shown at phases 0.681 and 0.682 at the top of Fig. 4. In view of these low S/N ratios, only spectral variations above a threshold of some 1 – 2% of the continuum could be considered statistically significant and suitable for (Z)DM. Do the Ni and the Cu lines meet these conditions, and what is the actual signal of the respective Ni and Cu spots postulated by LU10?

Figs. 2 and 4 give the answer. Adopting the stellar parameters listed by LU10 and assuming a Cu spot at 20° latitude with a radius of 20°, we modelled the variations of the blend containing the Cu II λ 5153.230 line, assuming solar abundances for the rest of the chemical elements. We consider 2 limiting cases: a "monolithic" spot with the maximum Cu abundance claimed by LU10 and the same spot with the minimum abundance. As it turns out, the spectrum variations due to the maximum spot are well below the 1% mark, the minimum spot is altogether undetectable. Looking at the same spectral interval, but modelling solely Ti (in a similar way to Cu), one sees small variations from both the minimum spot and the maximum spot, both of the order of the photon noise. Repeating this exercise for Cr, the minimum spot is undetectable, whereas the signal of the maximum spot is quite pronounced. The Fe contribution to the blend behaves similarly, but with more than 1 line involved. It is important to keep in mind that in the Doppler mapping procedure that has led to the published Ti and Cu maps, both the contributions of Cr and of Fe to the blend have been modelled with lines found elsewhere in the spectrum. The λ 5510 blend reveals equally interesting insight into what can be detected and what cannot even be guessed at. Both the minimum and the maximum Ni spots for example leave a signal which reaches a few 10^{-3} of the continuum at most and which is completely swamped by the signatures of the respective Cr, Fe and Y maximum spots. Not a single minimum spot of the 4 elements can be distinguished from the solar abundance stellar surface. Even the most cursory glance at the observed spectra at phases 0.681 and 0.682 will convince the experienced spectroscopist that modelling of the blend must be a very messy endeavour.

Let us go even further in this analysis and in our conclusions. Though in LU10 it is stated that *"it is possible to simultaneously calculate abundance maps of several chemical elements even from blended spectral lines"* this statement deserves some qualification in view of the strong interdependence between the various maps. We begin with the Fe map which is derived from 4 lines in a noisy spectrum, with 3 of these lines forming a blend at λ 5400 (see table 6 of LU10). This Fe map is used in the modelling of the λ 5510 blend, resulting – together with 2 more Cr lines – in a Cr map, but at the same time also in a Ni map and in a Y map, the latter together with 1 more Y line which in turn is blended with Fe. The Fe map and the Cr map (which depends to some degree on the Fe and the Y maps) have to be taken into account for the mapping of Ti and of Cu. Unless one were prepared to assume that a perfect fit to the Cr and the Fe profiles could be obtained at the 0.1% level, any signal apparently originating from spatial inhomogeneities in Ni or Cu must of necessity be dominated by errors in the Cr and Fe maps, resulting in entirely spurious maps. Given the marginal signature of minimum and maximum Ti spots, little faith can be placed in the corresponding map. The same holds probably true for Y. How far the maps of Cr and of Fe reflect the intrinsic abundances cannot be said at any reasonable level of certainty.

4. An extensively tested inversion code

Before addressing the performance of CossamDoppler, it is instructive to have a look back at the papers obtained after 2002, based on the various Invers codes, and to note the puzzling absence of realistic tests and/or cross-checks in all these inversions. Despite the fact that the complexity of their test cases did not go further than 3 large, well distributed, high-contrast chemical spots and fairly smooth dipole-quadrupole field geometries, KP02 have made the following key statement: *"We believe that the code can be successfully applied to the imaging of global stellar magnetic fields and abundance distributions of an arbitrary complexity."*. This firmly entrenched belief has to seen in conjunction with the opinion expressed by Kochukhov et al. (2012) on our CossamDoppler code: *"...one can see that the authors have failed to recover the true distribution of abundances spots even when they used a constant magnetic field and adopted the same mean atmospheric structure for calculations of the input spectra and for the inversion. A major discrepancy between the input and reconstructed abundance maps revealed in such a simple test indicates serious problems with the DI code used by S12. Evidently, results based on the application of this untested inversion code must be considered with caution."* There is only one conclusion that can be drawn, i.e. the idea that even under favourable circumstances, (Z)DM could fail to recover the correct abundance maps has been systematically rejected by O. Kochukhov and his co-authors.

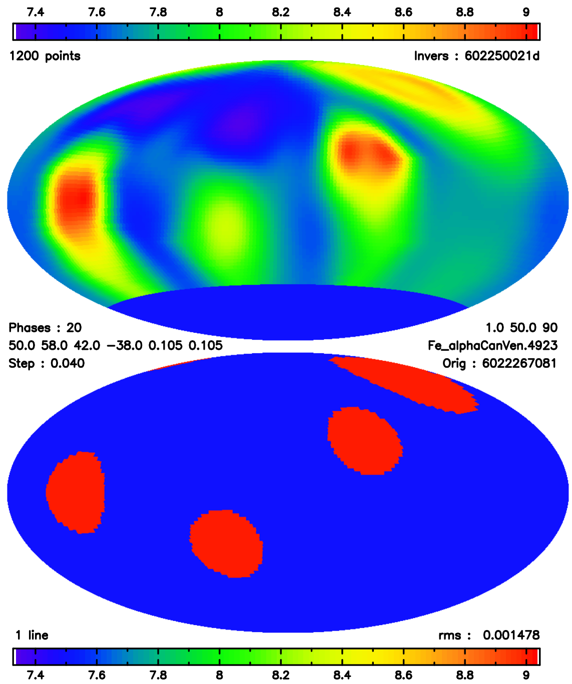

Fig. 5. Equal-area Hammer projection of a 4-spot Doppler mapping test case. The bottom part **(a)** of the plot shows the adopted spot distribution and contrast, the upper part **(b)** the result of the (Z)DM inversion. The spectral line used is the Fe II λ 4923.93 line at 20 equidistant rotational phases; the spectral resolution is 50 mÅ, giving an overall 1200 zero noise "observational" points to be used in the inversion. The inclination i of the rotational axis is 50°, the magnetic field originating from a tilted eccentric dipole is characterised by an obliquity of 46.3° and by a displacement from the centre of 0.148 (in units of radius). The rotational velocity is 50 km s^{-1}. All stellar and magnetic field parameters are assumed to be exactly known for (Z)DM. The residual rms error of the fit to the line profiles points is 1.5 10^{-3}. Note: Hammer projections show the whole stellar surface from −90° to +90°; the part invisible to the observer is clearly marked in the upper panel.

To demonstrate the excellent capabilities of the CossamDoppler code we deliberately chose the abundance distribution adopted by Kochukhov (2014a) (= K14), consisting of 4 well distributed "monolithic" high-contrast spots of 20° radius, rather than the three-spot geometry presented by KP02. The reason for this choice lies in the fact that on p. 6 of K14, one encounters the only instance in living memory of a published Doppler map which starkly exposes the limitations inherent in single-line inversions: despite the assumption that all the stellar parameters are exactly known, only 3 out of 4 spots are recovered, in contrast to the better known inversion tests presented in KP02. We adopted the same inclination, the same rotational velocity, the same 20 equidistant phases and the same spectral resolution as in K14, using the single Fe II line at 4923.93 Å. Fig. 5 shows an equal-area Hammer projection of both the initial map **(a)** and the result of the inversion **(b)**; as stated above, the magnetic field geometry, the inclination and the atmospheric model were assumed to be exactly known. There can be no doubt that the algorithm works perfectly well and that the results compare most favourably to those shown in K14. In the inversion, all 4 spots are well recovered as to the positions, even the southernmost spot. For 2 spots, the derived maximum abundances are close to the input data, for the outlying spots near the north pole and near the equator the abundances are less in agreement. A certain amount of smearing of the contours of the spots is visible in fig. 5, combined with extensions of the spots towards the southern hemisphere. Similar imperfections are also readily discernible in Figs. 5 and 9 of KP02, even more clearly so on p. 40 of Kochukhov (2014b), and likewise on p. 6 of K14.

CossamDoppler was also highly successfully applied to the three-spot KP02 test case. using the same two Fe II lines at $\lambda\,6147.74$ and $\lambda\,6149.26$. Thanks to hundreds of further tests with a large number of different horizontal abundance distributions, a couple of inclinations, various magnetic field geometries, and line lists containing from 1 to 20 lines, CossamDoppler certainly rivals any other (Z)DM code as to the variety and complexity of the test abundance maps that have been inverted. More or less sophisticated magnetic field geometries adopted in our tests are only of some interest when they provide additional information on the abundances via the QUV profiles since we always assume the magnetic field to be exactly known, a situation extremely favourable to abundance mapping, though unknown of in real life. Let us emphasise that unlike KP02, we have also extensively looked at spots featuring under-abundances!

It cannot be stressed enough that the correct algorithmic working of CossamDoppler or of any other (Z)DM code does not imply that it has to recover – even in the most approximate way – arbitrary input horizontal abundance distributions. As long as there is no mathematical proof that with all 4 Stokes parameters and high S/N ratio spectra you will invariably and of necessity correctly recover any complex abundance distribution, the only behaviour that is required for a (Z)DM code is the proper convergence towards a good fit to the "observed" profiles and the smoothest abundance structure compatible with the data, quantified either by maximum entropy or by Tikhonov regularisation. In Fig. 5 and Fig. 10c we show that CossamDoppler brilliantly meets these requirements, even in considerably more complex cases than those published by KP02. Let us add that nowhere in the literature do we find systematic tests with the Invers family of codes that would explore a huge parameter space with widely differing inclinations, rotational velocities and numbers of spots with different contrast. There is thus no statistically sound basis on which to build the belief that Invers "*can be successfully applied to the imaging of global stellar magnetic fields and abundance distributions of an arbitrary complexity.*". However, even if further tests had shown that in several instances the Invers codes failed to recover the correct abundance maps, this would not imply that a fundamental flaw in the (Z)DM algorithm had been laid bare. It could simply mean that the inverse problem at hand does not admit of a unique solution.

We therefore contend that when Rusomarov et al. (2015) claim that they have not found narrow belts of enhanced metals around the magnetic equator as predicted by Alecian & Stift (2010), this is irrelevant as long as they have not presented the proof that their code can recover this kind of (warped) narrow belts. One should always be aware that even the most beautiful maps of real stars constitute by no means a proof that the (Z)DM procedure has converged to the correct solution.

5. Neglected magnetic fields and spurious maps

Beauty is an important concept in mathematics and physics – see for example Chandrasekhar (1979) or Dirac (1939). What is beautiful, appealing to our aesthetic senses, is often thought to be true, but unfortunately this principle is frequently not only applied to the laws of nature but also to the presentation of empirical results. So when zero-field inversions are published with high-quality colour images, people are willing to consider them correct, discarding the findings of Stift (1996) who showed that in (Z)DM, the adoption of an incorrect magnetic field geometry leads to spurious abundance structure, even more so when the field is neglected altogether. Let us at this point recall that the resulting horizontally non-homogeneous abundances are due to the effect of "magnetic intensification" (Stift & Leone 2003) – i.e. the splitting and ensuing desaturation of the spectral lines – but that these abundances are not directly related to the magnetic field strength and/or orientation. Rather they represent the unpredictable (because entirely unphysical) response of the regularisation function to the Zeeman splitting and the local line profiles.

With CossamDoppler and with computing power vastly superior to what was available 2 decades ago, we decided to look again at the problem of zero-field inversions, and to illustrate the surprising plethora of apparent abundance structure that emerges – when the magnetic field is neglected – from Doppler mapping of a star featuring no spots. For this purpose, we have chosen the well-established field geometry of HD 154708 (Stift et al. 2013), thus eschewing possible criticism of using unrealistic magnetic parameters not to be found in a real star. Line profiles of 4 iron lines with different Zeeman patterns have been calculated for various field strengths and rotational velocities, but always with strictly the same geometry (for the definition of the non-axisymmetric tilted eccentric dipole model see Stift (1975)), assuming a homogeneous iron abundance of [Fe] = 8.00 (on a scale with [H] = 12.00).

Fig. 6**(a)** shows the abundance map obtained from the Fe II $\lambda\,4128.748$ line which splits into 12 sub-components, synthesised with $v \sin i = 17\,\mathrm{km\,s^{-1}}$ and a field ranging from 335 to 1290 G resulting from a tilted dipole with 48.3° obliquity and an offset of 0.148 (in units of radius) from the centre of the star. The angle between line-of-sight and the rotational axis is $i = 60°$. As explained above, it is the regularisation function that forces moderate under-abundances near one pole and moderate over-abundances (both about 0.2 dex) towards the equator. The fit to the "observed" profiles can be considered perfect (rms scatter of $3.5\,10^{-5}$), much better than what one could ever hope to achieve with real spectra. A somewhat different map **(b)** is obtained from the Fe II $\lambda\,4177.692$ line, split into 18 Zeeman sub-components, with the field almost 3 times larger than in the previous case, but with the same magnetic geometry. The star rotates at $v \sin i = 24\,\mathrm{km\,s^{-1}}$ and the inclination is $i = 75°$. The respective extensions and structures of the spots have changed and the range in spurious abundances is substantially larger than before, ranging from 7.91 to 8.56. Most disturbingly, exactly the same field strength and magnetic geometry lead to still another map **(c)** when the inversion is based on the Fe II $\lambda\,4258.154$ line split into 10 sub-components, with inclination $i = 60°$ and rotational velocity $v \sin i = 35\,\mathrm{km\,s^{-1}}$. Substantial over-abundances

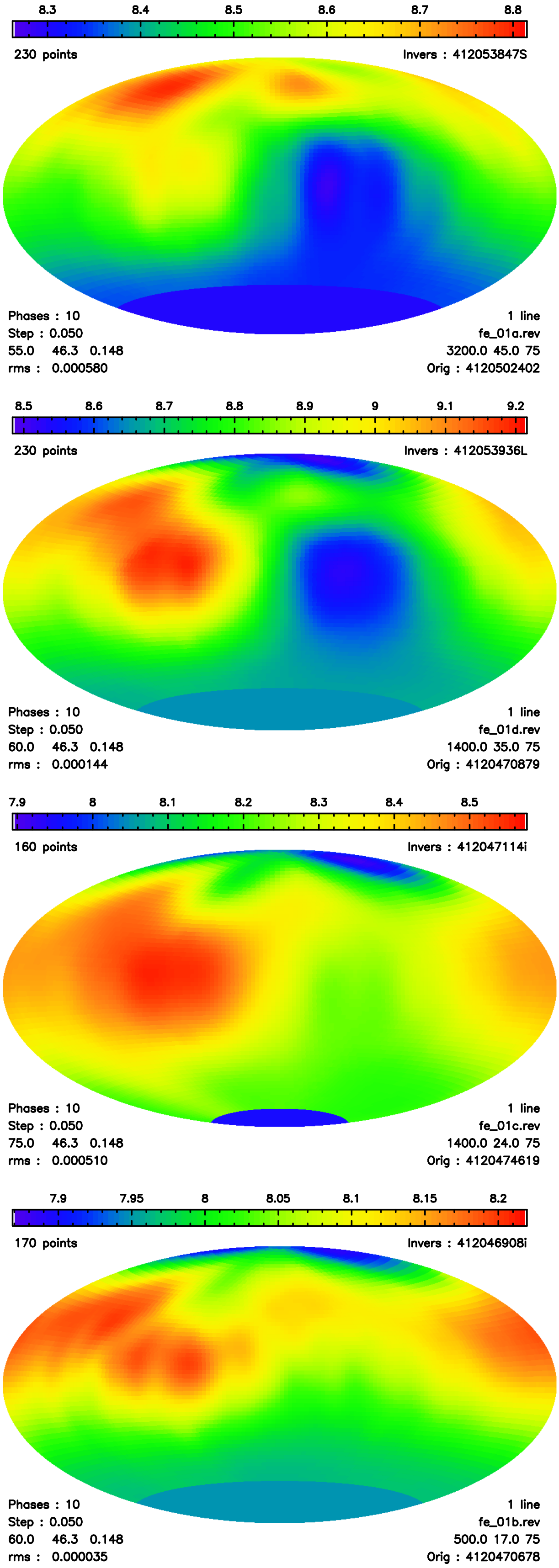

Fig. 6. Spurious ZDM maps obtained by neglecting the magnetic field of stars featuring no spots. The magnetic geometry adopted for calculating the "observed" profiles is the same for all 4 plots; field strength, inclination and rotational velocity differ as do the Zeeman splittings of the lines used.

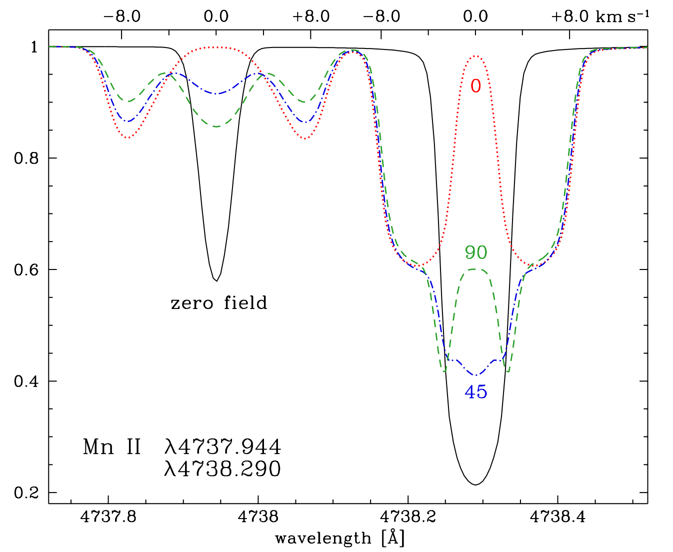

Fig. 7. Zeeman splitting of the Mn II lines at $\lambda\,4737.944$ and $\lambda\,4738.290$ in a 7 kG magnetic field. Shown are the cases of a longitudinal field (dotted profile), a transverse field (dashed) and the intermediate case of 45° (dash-dotted). The full line gives the unsplit lines. For illustrative purposes we have also given velocities with respect to the unsplit positions of the two lines.

are found all over the star, ranging from 0.5 dex to 1.2 dex. That the trend with magnetic field strength is entirely unpredictable becomes obvious from the map at the top **(d)**. For this particular simple Zeeman triplet (Fe I $\lambda\,4063.594$), with $i = 55°$ and $v \sin i = 45\,\mathrm{km\,s^{-1}}$, the spurious spots reach a contrast of more than 0.5 dex, the apparent over-abundances attain 0.8 dex and the map too again changes substantially. It has to be stressed that in all 4 examples shown, the fits to the "observed" profiles can for all practical purposes be considered perfect.

5.1. HD 3980 : a worst-case non-magnetic Doppler mapping scenario

The analysis of HD 3980 by NL is arguably the most extreme case found in the literature of a Doppler mapping procedure that completely neglects a strong magnetic field; it therefore constitutes a natural choice for a reassessment of such empirical zero-field maps. To start with, we want to point out the fallacy of the *a posteriori* effort made by NL to justify their zero-field approach, viz. by arguing with the limited impact of the magnetic field on *equivalent widths*. Be aware that this is in open violation of the very foundations of (Z)DM and that NL try to make us forget that (Z)DM is all about wavelength shifts due to rotation and magnetic splitting. The exact *shapes* of the local line profiles have to be known, not the *equivalent widths*, which are in no way part of the ZDM technique. What is puzzling: one would expect any person with access to a Zeeman Doppler mapping code to take the non-magnetic maps, switch on the magnetic field in the code and simply carry out a forward synthesis in order to quantify the effect of the magnetic field on the profiles. Instead, NL have resorted to line synthesis runs with the SYNTH3 and the SYNTHMAG codes, comparing non-magnetic and magnetic *equivalent widths* of lines of Fe, Cr, Eu and Gd in a vain effort to justify their zero-field analysis.

A close look at the two Mn II lines used by NL in a field of $B = 7$ kG reveals large Zeeman splittings as shown in Fig. 7 (a similar most instructive plot for all four Stokes parameters can

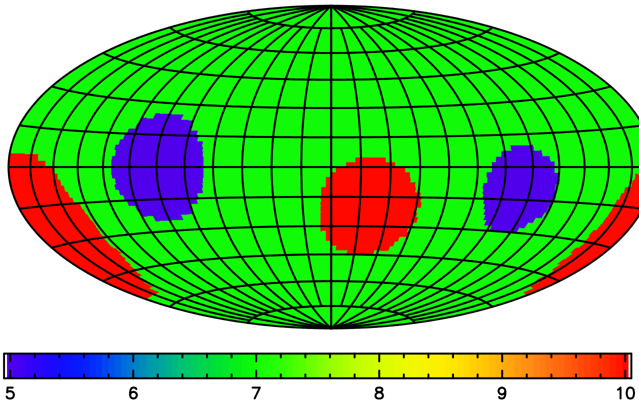

Fig. 8. The distribution of the manganese spots (red and blue) and of the oxygen spots (red only) in a simple model of HD 3980. The magnetic poles lie in the equatorial plane at the centre of the Hammer equal-area projection, and at the edges respectively.

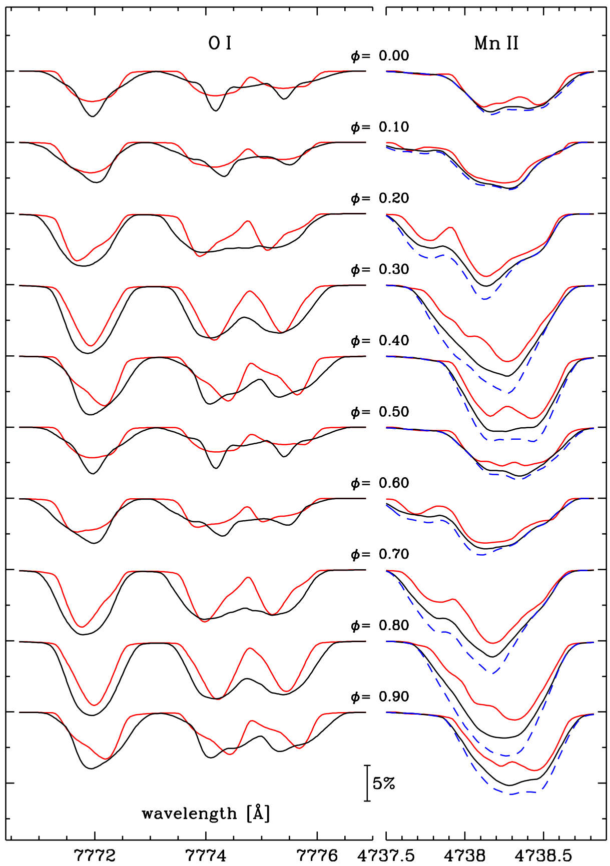

Fig. 9. Left: predicted phase-dependent profiles of the oxygen triplet for the simple model of HD 3980; right: predicted profiles of the manganese doublet. The profiles in black have been calculated with the magnetic field postulated by NL, with a polar field strength of B_p = 7 kG. The red profiles pertain to a zero-field synthesis. If appropriate metallicity-dependent local atmospheres were used in the line synthesis as postulated by Stift et al. (2012), one would arrive at the dashed blue profiles (only shown for Mn).

be found on p. 29 of Kochukhov 2014b). At a given wavelength, intensity values can differ hugely between the unsplit lines and the Zeeman-split lines at various field angles. In the longitudinal field case, intensities at the respective positions of the unperturbed lines attain a level very close to the continuum. Conversely, at ± 5-6 km s^{-1} from the centre of the λ 4738.290 line – where normally the line disappears in the continuum – substantial opacity comes from the σ-components. The λ 4737.944 behaves similarly but displays a larger Zeeman splitting; maximum opacity from the σ-components is now found at ± 7-8 km s^{-1}. In a zero-field inversion, the (Z)DM algorithm must interpret the missing signal from the line centre as a sign of an extremely low abundance, the signal from the σ-components as coming from spots. Confusingly enough, these entirely spurious spots would be located at different positions for the two lines. This leads us to the conclusion that zero-field inversions for stars with strong magnetic fields like HD 3980 must needs fail miserably.

It is not difficult to verify this conjecture. Approximating the published positions and major Mn abundance spots (Fig. 8) and adopting the suggested magnetic field parameters, the effect of the magnetic field on the Stokes I profiles can be modelled in a straightforward way. The results for 10 different phases are displayed in the right panel of Fig. 9. Over more than half the rotation period, large discrepancies are found between the non-magnetic profiles and the profiles calculated with the field of the dipole lying in the equatorial plane. If in addition, we apply an estimate of the impact of the huge over-abundances of elements like Si, Cr, Fe, Mn on the local atmospheric structure – for a more detailed discussion of the latter problem see Stift et al. (2012) – the discrepancies become even larger. More striking still are the results obtained for the oxygen triplet $\lambda\lambda$ 7772 – 7775 shown in the left panel of Fig. 9. In the light of all these profiles, it comes as no surprise that the abundances of oxygen (equal to hydrogen) and manganese (equal to helium) determined by NL are so extreme; their spurious nature is beyond reasonable doubt. The abundance patterns derived by means of zero-field Doppler mapping of this strongly magnetic star and their lack of correlation with the predictions of diffusion theory as applied to ApBp stars can therefore certainly not be used as an argument in favour of an alleged "lack of up-to-date theoretical models."

In this context we want to mention the zero-field results published for HR 3831 (Kochukhov et al. 2004a), another star with a substantial magnetic field of about B_p = 2.5 kG. As in the case of HD 50773, the maps have been extensively presented in lectures and reviews. One of the most striking characteristics of these maps are the incredibly large abundance contrasts shown for a number of chemical elements, reaching 7 dex for Ba, 6.3 dex for Na, 6.2 dex for Pr, 6.1 dex for Mn and Y, and between 4 and 5.5 dex for a couple of other metals. Incidentally, the maps for these 5 elements with the largest contrasts are all based on single-line or dual-line inversions. Since no star with magnetic field and abundances determined simultaneously has ever been shown to exhibit such extravagant behaviour, it is not unreasonable to surmise that the neglected magnetic field is at least partly responsible for the strikingly high-contrast abundance structures of HR 3831. Let us point out that differences between observed and predicted profiles can reach and even exceed 5% of the continuum intensity in several lines of the elements C, Mg, Si and Na, the fit to the λ 5895.92 of Na being of a particularly poor quality. It is surprising and puzzling that in their discussion of the HR 3831 results, Kochukhov et al. (2004a) insisted on the *"numerous examples of surface patterns which do not follow the symmetry of the dipolar magnetic topology"* although on the other hand they openly stated that they were *"interested in estimating the parameters of the simplest dipolar magnetic geometry and will not consider combinations of dipole and quadrupole components, since these complex topologies are poorly constrained by the available magnetic observables"*. In view of these major shortcomings in the establishment of the magnetic geometry of HR 3831 and because the effect of the magnetic field on the abundance maps has been completely neglected, we conclude that their study does not provide any useful constraints on theoretical models.

6. In a single line we trust?

Criticism has been voiced by Kochukhov et al. (2012) against Stift et al. (2012), alleging that in their article, these authors had not considered the '*"profoundly multi-line, multi-phase character of modern MDI"*. Such criticism can easily be shown to be unfounded since even a cursory survey of the relevant literature brings to light a large number of inversion based on a mere 1 or 2 spectral lines. In all, at least 35 maps have been published that have been derived from 1 line only of the element in question; at least 22 more Doppler maps rely on an analysis of 2 lines. In HD 3980 for example (Nesvacil et al. 2012), the respective maps of La, Eu, Ca, Nd and Pr are based on 1 spectral line, those of Mn, Gd, Fe, and Si on 2 lines. Lüftinger et al. (2010a) have used 1 line for Mg, Ti, Ni, and Cu, 2 lines for Ca and Y. Kochukhov et al. (2004b) have been basing their maps of C, Ca, Mg, Na, Ti, Y, Ba, Nd and Eu on 1 line, for Li, Si, Mn, Fe and Pr they used 2 lines. In Lüftinger et al. (2010b) Mg, Ca, Sc, Cr, Pr, Gd and Tb have been analysed with 1 line each, Ti, Ni and Y with 2 lines. For 53 Cam, Kochukhov et al. (2004a) have derived their Nd map from a single line, the maps for Si, Ca, and Ti from 2 lines each; 3 independent abundance maps have been determined for iron, each based on one single line. Since both tiny Fe spots and several regions depleted in Fe star exhibit sizes that range between a mere 1/5 and 1/3 of the KP02 test spot sizes, it would certainly constitute a fatal mistake to assume that the reliability of these single-line or dual-line maps matches those given by KP02 (with a triad of large "monolithic" spots, well distributed over the star, and exhibiting a high abundance contrast relative to the rest of the surface). The reader should keep in mind that even under favourable circumstances, spots near the invisible part of the stellar surface might be missed as in Kochukhov (2014a).

All of this means that there are not so many multi-line maps to be found in the literature and that the reliability of single-line inversions is not guaranteed at all. Claims concerning the non-existence of equatorial belts of over-abundances (as predicted by theory for some elements) which rely on single-line and/or zero-field inversions are to be eyed with suspicion. Finally it is important to have a critical look at the choice of lines used in the inversions. In none of at least 9 important articles on ZDM applied to ApBp stars has the iron map been derived with the help of the Fe ii lines employed in the KP02 tests. Having seen in the preceding section that the spurious maps resulting from zero-field inversions strongly depend on the spectral lines used, we decided to consider several different Fe line lists for our investigations, among them in particular the list published by Kochukhov & Wade (2010) for α^2CVn but also the three lines given by Kochukhov et al. (2004a) for 53 Cam.

Fig. 10(**a**) shows a fairly simple configuration of 3 extended spots: in 2 spots, abundances increase towards the centre, in 1 spot the abundance decreases. Using only the Stokes I profile of the Fe ii λ4923.93 line – employed in the mapping of α^2CVn – one obtains a map that can hardly be called a full success (**b**). Even though the 3 spots are recovered in some approximate form, shapes and positions relate rather poorly to those of the input model. Map (**c**), based on all 4 Stokes parameters, seems much more satisfactory, but still does not take into account the fact that the local atmosphere changes substantially with metal abundances – in particular of Fe and Cr – as pointed

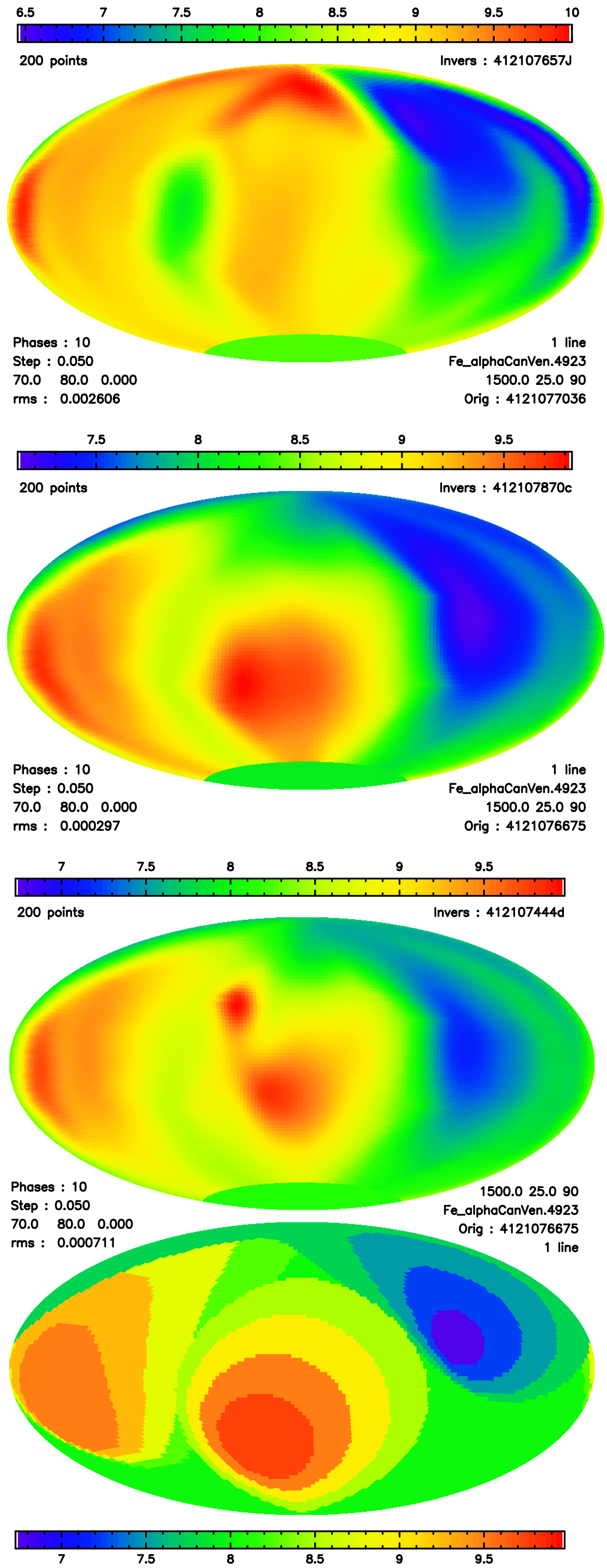

Fig. 10. ZDM maps from inversions based on the Fe ii λ4923.93 line (50 mÅ resolution, $i = 70°$, $v \sin i = 25\,\mathrm{km\,s^{-1}}$, centred dipole with 80° obliquity, polar field strength $B_p = 3$ kG). At the bottom **(a)** we show the adopted distribution of 3 large spots with inner structure, the map just above **(b)** gives the result of an inversion based on Stokes I only. Map **(c)** results from an inversion with all 4 Stokes $IQUV$ parameters. When the phase-dependent spectra are calculated with the correct local atmospheres, but a constant mean atmosphere all over the star is adopted in the inversion, we get the map displayed at the top (**d**).

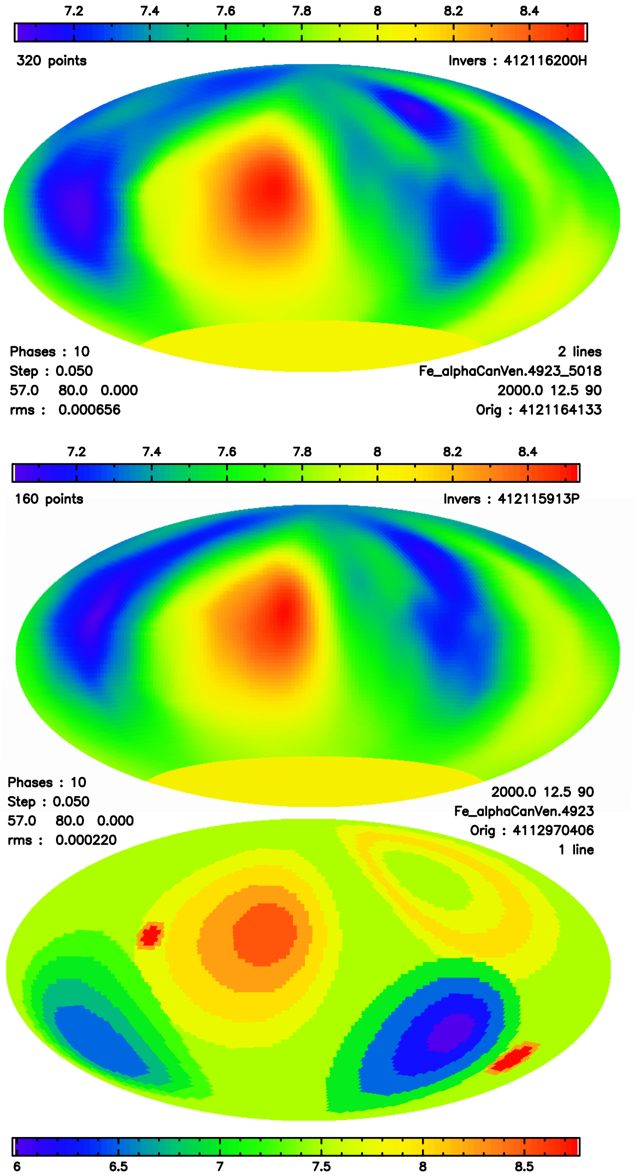

Fig. 11. ZDM maps from inversions based on the Fe II λ 4923.93 and λ 5018.44 lines, At the bottom **(a)** we show the adopted abundance distribution characterised by 1 ring-like feature and 5 spots of varying extension and structure. The map just above **(b)** gives the map based on the Stokes I profiles of λ 4923.93 only. The top map **(c)** results from an inversion using all 4 Stokes $IQUV$ parameters of both Fe II lines.

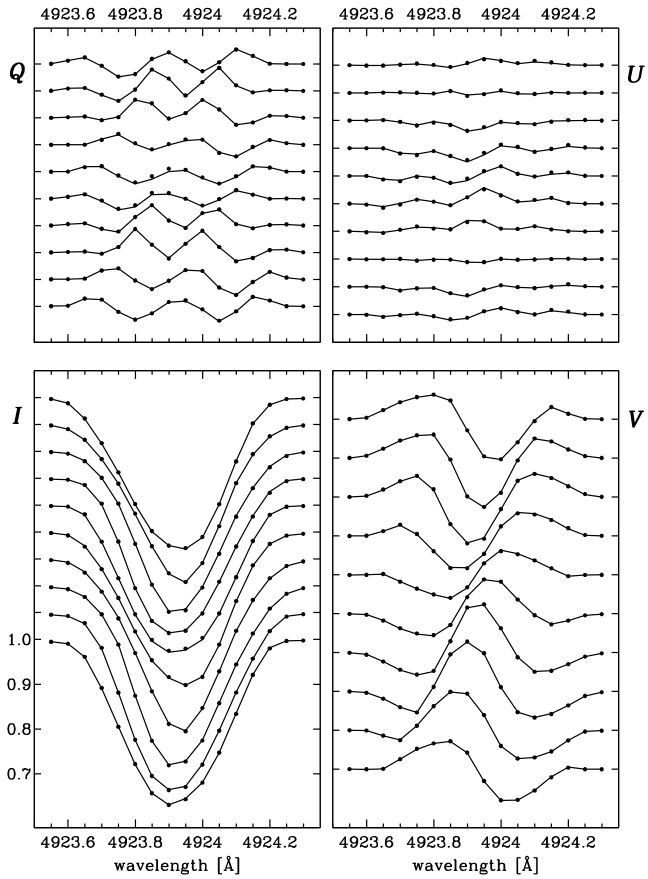

Fig. 12. Fit to Stokes $IQUV$ profile variations of the Fe II λ 4923.93 line. The filled circles have been calculated with the surface abundance structure displayed in fig. 11a, the lines represent the best fit to these "observed" profiles and are based on the abundance map shown in fig. 11c. The latter map is clearly characterised by a higher entropy than the original map. The profiles are offset by 6% in I, 3% in Q and U, and by 8% in V.

out by Stift et al. (2012). When the phase-dependent spectra are correctly calculated with the appropriate local atmospheres, and when in the inversion a constant mean atmosphere all over the star is adopted (as in essentially all inversions published so far) we arrive at the map shown at the top **(d)**. This map bears very little resemblance to the original input map; the central strong spot disappears and one finds a strange high-abundance feature emerging near the northern pole.

Finally, we looked at a more complex abundance map consisting of 5 spots and one ring-like structure (Fig. 11). Not unexpectedly, the Stokes I only inversion with the Fe II λ 4923.93 line fails completely, recovering just the position of the central spot. Adding the λ 5018.44 line and using all four Stokes parameters, the number of available profile points to be fitted is multiplied by a factor of 8 but to no avail. The map only changes a wee little bit and still no more than 1 or 2 of the 6 abundance structures in the input model can be approximately recovered! Once again we point out the excellent fit to the "observed" Stokes profiles of both the Fe II λ 4923.93 and the λ 5018.44 lines, with an rms scatter of $6.6\,10^{-4}$. When real stellar spectra are inverted, there is absolutely no way to ascertain the nature of the resulting maps, whether they represent the true horizontal abundance inhomogeneities or whether they are entirely spurious. The best fit to the data, subject to the restrictions imposed by the regularisation function, is the decisive criterion. So whoever got results as displayed in fig. 12 from an unknown stellar source would not hesitate to adopt this solution. To put it succinctly: the fact that two completely different abundance maps lead to the same perfect fit to all 4 noise-free Stokes profiles of 2 unblended spectral lines (with precisely known atomic parameters) in a star with exactly known physical parameters and exactly known magnetic geometry unequivocally demonstrates the existence of multiple solutions to the (Z)DM problem.

7. Enter diffusion theory

The statement that appropriate local atmospheres have to be used in the proper modelling of magnetic Ap stars with pronounced horizontal abundance inhomogeneities has to be elaborated upon in view of the results on atomic diffusion in magnetic fields presented by Alecian (2015) and by Stift & Alecian (2016). First

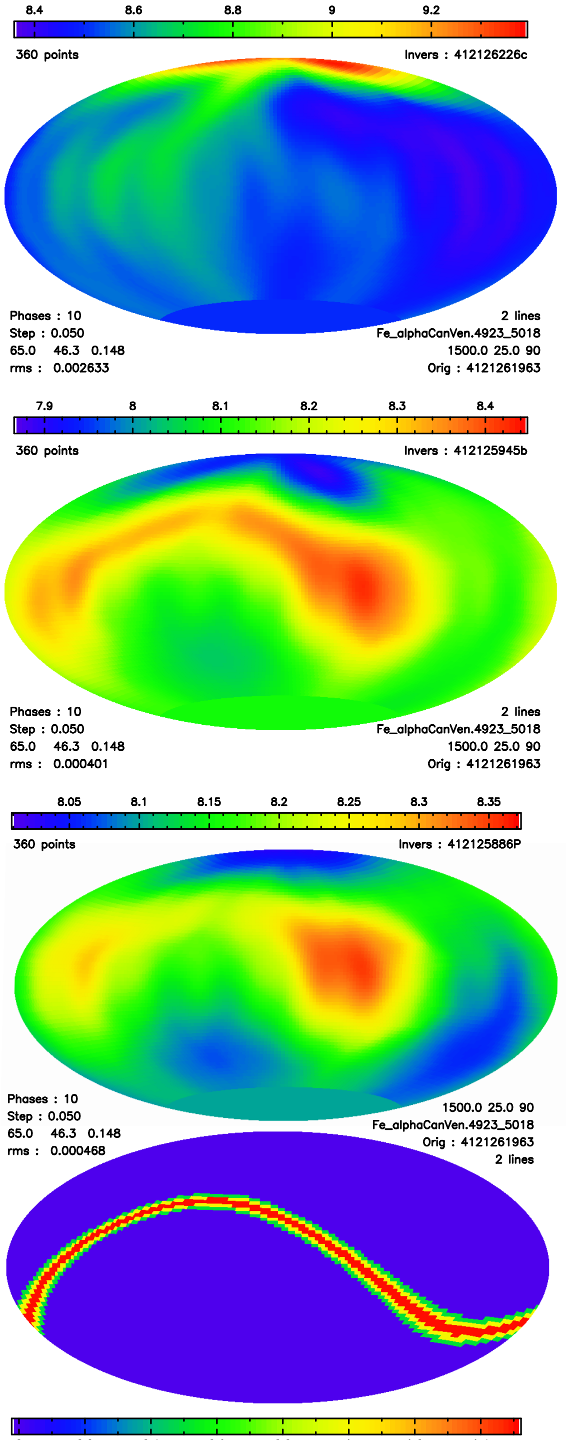

Fig. 13. ZDM inversions based on the two Fe II lines at λ 4923.93 and λ 5018.44. A warped ring of enhanced abundances which follows the magnetic equator is shown at the bottom **(a)**. The map just above **(b)** represents the ZDM result based on Stokes *I* profiles only. Map **(c)** has been derived with the help of all 4 Stokes parameters. In both cases the stellar and magnetic field parameters have been assumed to be exactly known. A zero field inversions yields the perfectly spurious map at the top **(d)**. Note the spectacular north polar spot.

of all, let us point out that not a single (Z)DM analysis so far has taken into account stratified abundances, although a number of papers have dealt with the alleged detection of (global) vertical abundance inhomogeneities in several ApBp stars – see the review by Ryabchikova (2008). It appears that the authors of these studies have quietly agreed on a spot/stratification dichotomy, i.e. the mutual exclusion of spots on the one hand, and of stratified abundances on the other hand. This means that in (Z)DM, abundances (and magnetic fields) are allowed to vary horizontally, but are assumed – at a given point on the surface – to be vertically constant from the bottom to the top of the atmosphere. Conversely, in the abundance analysis of stars like γ Equ or β CrB, the assumed absence of spots is accompanied by stratification profiles that are constant all over the star, regardless of the local magnetic field vector. One of the few examples of a star having been subjected to analyses according to both these scenarios is HD 24712; Lüftinger et al. (2004) found a 2 dex jump in the *global vertical* Fe distribution, whereas the *horizontal* contrast in the unstratified ZDM analysis by Lüftinger et al. (2010b) is limited to about 0.6 dex. Under-abundances relative to the solar value range from -0.1 dex to -0.7 dex. Can these apparently conflicting results be explained within the framework of diffusion theory?

May we draw at this point attention to a consistently overlooked caveat in the paper by Lüftinger et al. (2010b). It says verbatim: *"It is still possible however that part of the horizontal abundance structure we find is due to variation of the chemical stratification profile across the stellar surface."* In fact, variations in the stratification profiles are exactly what the time-dependent simulations of the diffusion of iron-peak elements predict. Figs. 3 and 5 of Stift & Alecian (2016) reveal a strong dependence of the Fe stratification on the angle of the magnetic field with respect to the surface normal, to a substantially lesser degree on the magnetic field strength. These authors also showed that, in contrast to accepted wisdom, even very moderate field strengths can lead to impressive over-abundances in high-lying atmospheric layers. Unfortunately, a thorough investigation – in the light of the predictions of diffusion theory – of the relation between spot based and global stratification based abundance analyses that would give us useful clues as how to interpret the diverging results has yet to be carried out.

7.1. Warped rings, a challenge for ZDM

So far, the literature on (Z)DM has dealt almost exclusively with the detection of spot-like abundance structures. For this kind of structure, it has been established in the preceding sections that abundance maps obtained by zero-field inversions of stars with fields of several kG can in no way be relied upon. There can be little doubt that this must also hold true for the rings about the magnetic equator predicted by Alecian & Stift (2010) and by results from time-dependent calculations presented by Stift & Alecian (2016). What is less clear is the answer to the question of how well ring-like abundance structures can be recovered when the magnetic field geometry, the field strength, and the angle between rotational axis and the observer are all exactly known. In that context one must not forget that when rings result from magnetic surface fields represented by the non-axisymmetric eccentric tilted dipole model, they will in general be warped. For a successful application of the eccentric tilted dipole to β CrB and to HD 126515 see Stift (1975) and Stift & Goossens (1991); HD 154708 has been the object of a more sophisticated study including the modelling of the profiles of near-infrared lines of Si (Stift et al. 2013).

To start with a simple idealised case, we assumed the magnetic geometry of HD 154708 and vertically constant over-abundances at the magnetic equator, decreasing rapidly with the angle between field vector and the surface normal as shown in Fig. 13**(a)**. The ZDM inversion based on the two iron lines

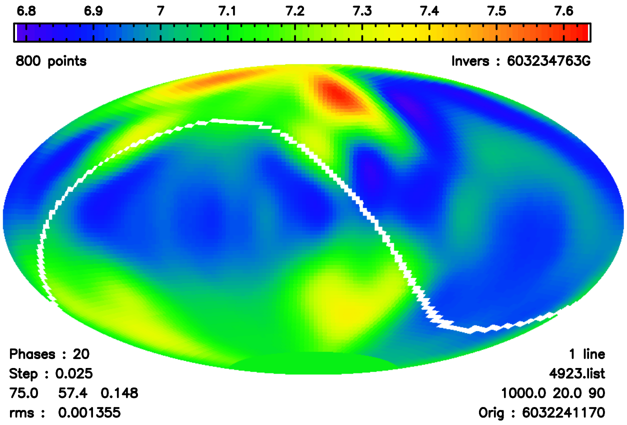

Fig. 14. Doppler map obtained from the inversion of the Fe II λ 4923.93 line, adopting a mean atmosphere with T_{eff} = 10000 K, log g = 4.0 and [Fe] = 8.0. The input spectrum has been calculated with field-dependent stratification profiles as shown in Stift & Alecian (2016); over-abundances decrease with distance from the magnetic equator (indicated as a white warped ring). The non-axisymmetric oblique rotator model is characterised by inclination i = 75°, obliquity β = 57.4° and dipole offset 0.148 (in units of stellar radius). Field strengths range from 660 G to 2375 G.

λ 4923.93 and λ 5018.44 yields a very unsatisfactory map **(b)** with a single dominating spot when Stokes I only is used. Please note the extremely reduced contrast! With all four Stokes parameters, the inversion **(c)** seems to recover at least part of the ring, which however is very much washed out, still suffers from a highly reduced contrast and rather looks like two spots connected by a kind of bridge. Near the pole there now appears a region exhibiting what looks like moderate under-abundances. Finally we establish an entirely spurious map **(d)** by carrying out a zero field inversion with the two lines given above. A spectacular north polar spot emerges with some wispy structure extending towards the southern pole. In contrast to the inversions where the magnetic field has been fully taken into account, the fit to the profiles is no longer perfect but definitively not worse than what has been achieved for HD 3980 by Nesvacil et al. (2012). The present idealised examples indicate that seemingly simple warped abundance rings can be difficult to recover (as are multiple spots as shown before) despite an exact knowledge of the magnetic field strength and geometry, particularly if the inversion relies on just 1 or 2 lines and if only Stokes I is used. The probability of obtaining the true abundance maps appears to increase with the use of all four Stokes parameters but even then it cannot be excluded that the inversion results in spurious maps.

The physical reality that emerges from time-dependent diffusion calculations makes life for the (Z)DM aficionado even more difficult. The abundances still depend on the angle between magnetic field vector and the surface normal but they can no longer be considered constant with depth. Abundances are stratified: over-abundances in the higher layers of the atmosphere can be accompanied by under-abundances in intermediate layers. So far this has not been taken into account in any of the numerous (Z)DM inversions, mean unstratified atmospheres being at the basis of every single map published. We thus decided to synthesise a spectrum resulting from field-dependent stratification profiles and to carry out an inversion with a mean unstratified atmosphere. For this purpose, we selected the Fe II λ 4923.93 line and we chose a magnetic geometry similar to the one determined for HD 154708. The resulting abundance map, based on a fit to all four Stokes parameters and with the effect of the magnetic field taken correctly into account, is shown in fig. 14. For illustrative purposes, we outline the magnetic equator; the plot makes it abundantly clear that the intrinsic correlation between magnetic field direction and abundances is not recovered.

8. A final caveat: (Z)DM results and stellar atmospheres

Stift et al. (2012) have drawn attention to the fact that according to Chandrasekhar (1935) and his deservedly famous "picket-fence" model, the structure of a stellar atmosphere at constant T_{eff} and log g varies with metallicity, and that this has to be taken into account in the context of (Z)DM. Such an approach has been considered a luxury by Kochukhov et al. (2012) who claimed the use of mean atmospheres to be legitimate. Their arguments were however in essence based on the study of α^2 CVn by Kochukhov & Wade (2010) and avoided the discussion of examples of more exotic over-abundances such as those found in κ Psc (Piskunov et al. 1998), ι Cas (Kuschnig et al. 1998), HR 3831 (Kochukhov et al. 2004a), 53 Cam (Piskunov 2008) and HD 3980 (Nesvacil et al. 2012). Although it would seem desirable – when the contrast of the over-abundant structures exceeds several dex as in the stars mentioned above – to ensure a certain degree of self-consistency of the (Z)DM analysis through the use of appropriate metallicity dependent local atmospheres, we are not prepared to reenter this discussion for a simple reason. We think that in Stift et al. (2012) we have just glimpsed the notorious "tip of the iceberg" when we realised the important role of appropriate local atmospheres. Pushing this analysis further, we are told by the work on equilibrium stratifications (Alecian 2015) and by the stationary solutions to the time-dependent problem (Stift & Alecian 2016) that vertically constant abundances are unlikely to exist in real ApBp stars. Fig. 14 proves that inversions based on mean unstratified atmospheres are likely to yield spurious maps. Another close look at the physics of stellar atmospheres however suggests that more fundamental problems are looming beyond this horizon. We contend that not only are the extreme abundance maps mentioned before unlikely to remain stable over short time-scales, much less over tens or thousands of years, but that stratifications in magnetic ApBp atmospheres cannot be determined by means of a 1D approach, i.e. by approximating them by isolated "cylinders" characterised by specific stratification profiles of the various elements and the corresponding local atmosphere. To arrive at this conclusion, it is not necessary to go beyond some most basic physical considerations which we intend to present in the following.

For solar abundances, the mean molecular weight of the gas is about μ = 1.26. Taking the abundance values shown in figs. 4-6 of NL, this becomes something like μ = 16, more than 10 times the solar value. The local pressure scale height inside the spot would be just 1/12 of the scale height of the surroundings, making the Wilson depression in sunspots almost vanishingly shallow in comparison. In order to visualise the kind of problem arising from such a configuration, we calculate model atmospheres corresponding to the "normal" atmosphere and to the spot with the extreme abundances. Since it defies the capabilities of most atmospheric codes to establish such unrealistic atmospheres, we restrict ourselves to the case of [Fe] = 10.50, with the other elements exhibiting solar abundances, resulting in μ = 2.8. Fig. 15 reveals the huge differences – which reach 4 orders of magnitude – in gas pressure at a given geometrical depth x (counted from the respective minimum optical depths of the atmospheric models). The greatly reduced scale height in the spot, 45% of

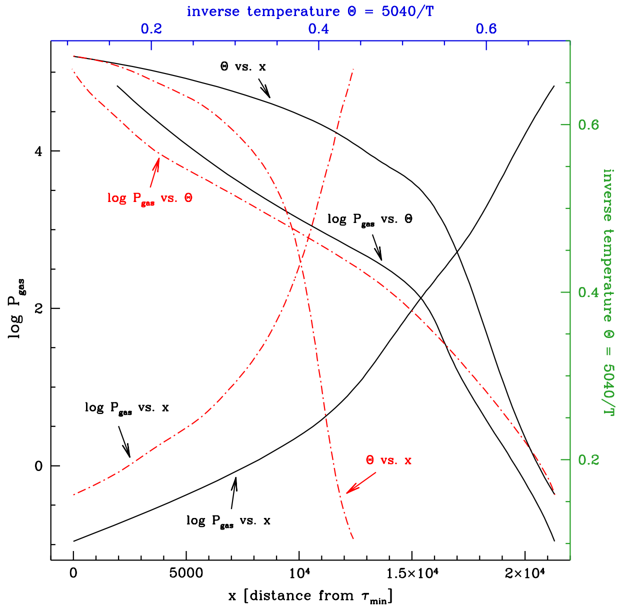

Fig. 15. Structure of two stellar atmospheres with identical $T_{\rm eff}$ = 12000 K and $\log g$ = 4.0, but different Fe abundances. The black full lines show the results for [Fe] = 7.50, the dash-dotted red lines for [Fe] = 10.50. The abundances of all other 91 elements are assumed to be solar.

the scale height outside, shows up clearly in the respective temperature vs. x relations. For illustrative purposes, we have also plotted the relations $P_{\rm gas}$ versus temperature which display noticeable differences.

In the absence of stabilising forces, a system consisting of a stellar spot and the "normal" atmosphere will establish horizontal pressure equilibrium on the dynamical time-scale, i.e. almost instantaneously with respect to the slow diffusive motions that lead to the build-up of vertical abundance inhomogeneities. Even the slightest pressure differences will immediately be ironed out by horizontal flows of material. Keeping in mind that the respective geometrical depth values in the spot and in the "normal" atmosphere do not refer to the same absolute zero point relative to the observer or to the centre of the star, we do not know whether the gas pressure in the spot will always be larger than outside the spot, or whether the 2 curves intersect at some layer, so that high up in the atmosphere low-abundance material from outside would dilute the high abundances inside the spot and deeper down just the opposite would hold true. All of this admits of one conclusion only: spots with extreme unstratified over-abundances, embedded in an atmosphere with very different unstratified abundances, cannot be stable. Both diffusion theory and (Z)DM will have to enter as yet uncharted territory.

One might object that strong vertical magnetic fields could stabilise the high-abundance "cylinders", much in the way this happens in sunspots. For several reasons we don't think that this is likely to work. Let us look first at observational evidence. Take for example 53 Cam and the incredible complexity of its magnetic field (Piskunov 2008) with field strengths ranging from 1.4 kG to 26.1 kG. The abundances and positions of the Fe spots with respect to the magnetic field defy simple explanations: both the largest over-abundance and the most extreme under-abundance are found at positions on the stellar disc that can be considered magnetic poles, featuring field strengths in excess of 20 kG. The spot with the second largest Fe over-abundance is located near the magnetic equator and there is no vertical field either to confine the strong Si spots which are also situated near the magnetic equator. How could one ever reconcile such strange spot behaviour with a stabilising magnetic field? Just to show that this problem is not unknown of in other stars, let us mention the studies of α^2 CVn by Kochukhov & Wade (2010) and by Silvester et al. (2014). In this star, there is nothing that would resemble a magnetic pole, rather are the regions with the strongest magnetic field (> 4 kG) located near to the magnetic equator. Neither the horizontal distribution of Cr nor that of Fe show any clear correlation with magnetic field strength or field direction but Fe seems to be enhanced near the magnetic equator, far from the strongest fields. On the theoretical side, it seems unlikely that even if there were a "cylinder" of extreme metallicity, confined in a vertical magnetic field, such a configuration would remain stable over years and decades, given the huge differences in pressure, density and scale height.

9. Conclusions

There is an apparent contradiction between theoretical models and the ever increasing number of (Zeeman) Doppler maps which have accumulated over recent years. Inversions which seem to provide unprecedented details of abundance distributions and which seem to demonstrate high levels of complexity in the surface structure have invariably been interpreted as just revealing the hopeless inadequacy of theoretical results. Thus the (Z)DM literature on ApBp stars published over the last decades is full of statements like "*This all suggests that important details are missing from the theory relating to the formation of horizontal abundance structures and the magnetic field*" (Silvester et al. 2014). Concentrating on the alleged few virtues and many failings of diffusion theory, none of the many authors working on ZDM has ever questioned the assumptions underlying the interpretation of ZDM results, viz. that abundances are unstratified, that mean stellar atmospheres are a good approximation to the local atmospheres, and that abundance maps are unique.

We do not pretend that the present-day status of diffusion theory and associated numerical modelling are unassailable or that the world is near to a full understanding of what happens in the atmospheres of magnetic ApBp stars. However, we think that any valid criticism of theoretical work on diffusion that argues with contrasting empirical results must needs be based on extensive and realistic tests. Such tests have to involve all well-known stellar atmospheric physics (e.g. the metallicity dependence of the temperature and pressure structure), they must not be restricted to spot-like abundance structures, they have to include stratification of the chemical elements – which are both predicted by theory and detected in stellar spectra. How is it possible that diffusion theory has been abundantly criticised for its alleged shortcomings but that not once up to the present day – apart perhaps from Stift (1996) – has anybody ever seriously questioned the validity of (Z)DM results based on mean atmospheres, unstratified abundances and the neglect of magnetic fields?

This paper finally partially remedies this situation by demonstrating unequivocally that even in fairly simple test cases, be they based on 3-7 spots that are not assumed monolithic, or on a warped ring following the magnetic equator in a tilted eccentric dipole model, there is no guarantee whatsoever that the surface abundance structure taken for input will be correctly recovered. This holds in particular for inversions using only a single spectral line in Stokes I, but unexpectedly, even with all four Stokes $IQUV$ parameters and/or more lines, a completely spurious abundance map can by no means be excluded. This comes somewhat as a surprise since in our tests we have always assumed the magnetic field strength and geometry to be *exactly* known (zero-field inversions of course excepted) and the dependence of the local atmosphere on the abundance – taken to be vertically constant – to be negligible. Any abundance map derived from 1 or 2 lines only must therefore be regarded with suspicion.

We have also shown that the application of zero-field inversions to strongly magnetic stars as for example carried out by Nesvacil et al. (2012) for HD 3980 cannot give correct results. The extreme over-abundances derived for HD 3980 – remember that in some spots manganese and oxygen are claimed to be as abundant as He, Si allegedly even as abundant as hydrogen – are without doubt entirely unphysical. Claims and conclusions based on this star or on empirical zero-field inversion abundance maps cannot constitute challenges to present-day diffusion theory, nor can the low-contrast Ni and Cu spots derived by Lüftinger et al. (2010a) from the remarkably noisy Narval spectra of HD 50773 provide "important observational constraints" for the modelling of radiative diffusion in magnetic stars.

9.1. A bleak outlook?

From the results discussed in the preceding sections it has emerged that (Z)DM has so far failed to provide us with really trustworthy empirical data that could serve as constraints to theory. It is also true that at present numerical modelling of atomic diffusion is not capable of predicting abundance maps and stratifications for a given star. Thanks however to tremendous progress made over recent years, it has been established incontrovertibly from time-dependent diffusion calculations that the build-up of vertical abundance structure is highly sensitive to the inclination of the magnetic field lines – even when fields are fairly moderate (Stift & Alecian 2016). This kind of modelling has unveiled the complex evolution of abundance stratifications and has provided new insight into the physics of this process. Since both equilibrium and time-dependent stationary stratifications have been found to depend strongly on the field direction, it becomes clear that (Z)DM analyses based on mean unstratified atmospheres and localised vertically constant over- and/or under-abundances are inadequate; the same holds for stratification analyses that assume the same vertical abundance profile all over the star.

One has to realise that suddenly there is a much larger number of free parameters to be determined with the help of (Z)DM. Instead of having to deal with a single abundance value for every surface element, an entire stratification profile has to be determined which in turn depends not only on the magnetic field strength and direction at a particular position, but also on the magnetic geometry of the surrounding surface elements. No regularisation procedure will be able to constrain the multitude of possible solutions – an attempt by Kochukhov et al. (2006) to derive global stratification profiles, based on vertical regularisation, has been shown by Stift et al. (2012) to be seriously flawed. In other words, only the introduction of physical constraints could possibly remove the indeterminacy of the ill-posed inverse problem, a kind of approach strongly advocated by Donati in 2001 at the conference on *Magnetic Fields Across the Hertzsprung-Russell Diagram*. Donati surmised that irrespective of the regularisation function used, data sets with all four Stokes parameters did not necessarily contain enough information on the field to accurately recover the magnetic distribution, even in simple cases. While, as stated at the beginning, we have not looked at the recovery of magnetic fields, we have shown that this non-uniqueness of solutions certainly applies to vertically constant abundance distributions despite the fact that there is only 1 unknown per surface element, compared to 3 unknowns per surface element when it comes to the magnetic field.

Unfortunately, as can be deduced from the discussion in section 8, physical constraints will be rather difficult to establish. It is certainly feasible to apply a truly multi-line approach with spectral lines that exhibit very different centre-to-limb behaviour as presented in Fig. 3 of Stift (1986) in order to maximise the diagnostic content of the simultaneously modelled separate Stokes $IQUV$ profiles. We can also afford to establish appropriate stratified local atmospheres so that there is no need to base inversion codes on mean atmospheres. Still, even when individual stratifications are available as predicted from theory for a grid of magnetic field angles and field strengths, there is no way to predict and parameterise the 3D abundance structure of an ApBp star permeated by a non-axisymmetric magnetic field with a warped magnetic equator. It would seem that (Z)DM has to be reinvented on a new and different basis.

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